Hydrodynamics at RHIC (ideal and viscous): Where it works, where and how it breaks down, and why.*

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Reference (review): UH, arXiv:0901.4355 [nucl-th] (Landolt-Börnstein)

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1. **Formalism**
2. **Numerical implementation and results**

Starting point: The conservation laws

$$\partial_\mu N^\mu = 0 \qquad \text{charge conservation}$$

$$\partial_\mu T^{\mu\nu} = 0 \qquad \text{energy-momentum conservation}$$

$$\partial_\mu S^\mu \geq 0 \qquad 2^{\text{nd}}\ \text{law of thermodynamics}$$

Ideal fluid decomposition

Ideal fluid dynamics $\Longleftrightarrow$ local thermal equilibrium $f(x,p) = f_{\rm eq}(x,p)$
$\Longleftrightarrow$ collision time scale $\ll$ macroscopic time scales $\Longleftrightarrow$ **strong coupling**

$$
\begin{aligned}
N^\mu &= \frac{1}{(2\pi)^3}\int \frac{d^3p}{E} p^\mu f(x,p) = n\,u^\mu & \qquad n &= \text{(net) charge density} \\
T^{\mu\nu} &= \frac{1}{(2\pi)^3}\int \frac{d^3p}{E} p^\mu p^\nu f(x,p) & e &= \text{energy density} \\
&= (e+p)\,u^\mu u^\nu - p\,g^{\mu\nu} & p &= \text{pressure} \\
&= e\,u^\mu u^\nu - p\,\Delta^{\mu\nu} & \Delta^{\mu\nu} &= g^{\mu\nu} - u^\mu u^\nu \\
S^\mu &= s\,u^\mu & s &= \text{entropy density}
\end{aligned}
$$

First law of thermodynamics: $\quad Ts = p - \mu n + e$

$$\partial_\mu N^\mu = \partial_\mu T^{\mu\nu} = 0 \Longrightarrow \partial_\mu S^\mu = 0$$

(in absence of shock discontinuities, entropy is conserved)

Ideal fluid equations (in comoving frame)

Convective and transverse derivative: $\partial_\mu = u^\mu D + \nabla^\mu$
$D \equiv u^\nu \partial_\nu, \quad \nabla^\mu \equiv \Delta^{\mu\nu}\partial_\nu$

$$\dot{n} = -n\,\theta$$

$$\dot{e} = -(e+p)\,\theta$$

$$\dot{u}^\mu = \frac{\nabla^\mu p}{e+p} = \frac{c_s^2}{1+c_s^2}\frac{\nabla^\mu e}{e}$$

$$p = p(n,e)$$

$\dot{f} = u^\mu \partial_\mu f \equiv Df =$ time derivative in local rest frame

$\theta \equiv \partial \cdot u =$ local expansion rate

$c_s^2(T) = \frac{\partial p}{\partial e} =$ (speed of sound)^2

equation of state (EOS)

6 equations for 6 unknowns: $n,\ e,\ p,\ u^\mu$

Non-ideal fluid decomposition

$$f(x,p) = f_{\rm eq}(x,p) + \delta f(x,p)$$

$$\boxed{\begin{aligned} N^\mu &= n\,u^\mu + V^\mu \\ &= N^\mu_{\rm eq} + \delta N^\mu \\ T^{\mu\nu} &= e\,u^\mu u^\nu - p\Delta^{\mu\nu} \\ &\quad - \Pi\Delta^{\mu\nu} + \pi^{\mu\nu} \\ &\quad + W^\mu u^\nu + W^\nu u^\mu \\ &= T^{\mu\nu}_{\rm eq} + \delta T^{\mu\nu} \\ S^\mu &= s\,u^\mu + \Phi^\mu \\ &= S^\mu_{\rm eq} + \delta S^\mu \end{aligned}}$$

$n = u_\mu N^\mu$

$V^\mu = \Delta^{\mu\nu} N_\nu =$ charge flow in l.r.f.

$e = u_\mu T^{\mu\nu} u_\nu$

$\Pi = -\frac{1}{3}\Delta_{\mu\nu}T^{\mu\nu} - p =$ viscous bulk pressure

$W^\mu = u^\nu T_{\nu\lambda}\Delta^{\lambda\mu} =$ energy flow in l.r.f.

$= q^\mu + \frac{e+p}{n} V^\mu \quad q^\mu =$ heat flow in l.r.f.

$\pi^{\mu\nu} = T^{\langle\mu\nu\rangle}$

$\equiv \left[\frac{1}{2}(\Delta^{\mu\sigma}\Delta^{\nu\tau} + \Delta^{\mu\tau}\Delta^{\nu\sigma}) - \frac{1}{3}\Delta^{\mu\nu}\Delta^{\tau\sigma}\right] T_{\tau\sigma}$

$=$ viscous shear pressure tensor $(\pi^\mu_\mu = 0)$

$s = u_\mu S^\mu$

$\Phi^\mu = \Delta^{\mu\nu} S_\nu =$ entropy flow in l.r.f.

Frame choice and matching conditions

The local equilibrium distribution $f_{\mathrm{eq}}(x,p)$ (with local temperature $T(x)$ and chemical potential $\mu(x)$) that best matches the non-equilibrium $f(x,p)$ is defined by the matching conditions

$$\boxed{u_\mu\, \delta T^{\mu\nu}\, u_\nu = u_\mu\, \delta N^\mu = 0}$$

Local rest frame is ambiguous:

Eckart frame: $u^\mu = \frac{N^\mu}{\sqrt{N\cdot N}}$: $\qquad V^\mu = 0,\;\; q^\mu = W^\mu$

Landau frame: $u^\mu = \frac{T^{\mu\nu} u_\nu}{\sqrt{u_\alpha T^{\alpha\beta} T_{\beta\gamma} u^\gamma}}$: $\qquad W^\mu = 0,\;\; q^\mu = -\frac{e+p}{n} V^\mu$

(Intermediate frames also possible.)

$\Longrightarrow$ Need $1+3+5=9$ additional equations for Π, q^μ, $\pi^{\mu\nu}$ from underlying transport theory.

Non-ideal fluid equations

$$
\begin{aligned}
\dot{n} &= -n\,\theta - \nabla\cdot V + V\cdot\dot{u} \\
\dot{e} &= -(e+p+\Pi)\,\theta + \pi_{\mu\nu}\sigma^{\mu\nu} - \nabla\cdot W + 2\,W\cdot\dot{u} \\
(e{+}p{+}\Pi)\,\dot{u}^\mu &= \nabla^\mu(p+\Pi) - \Delta^{\mu\nu}\nabla^\sigma\pi_{\nu\sigma} + \pi^{\mu\nu}\dot{u}_\nu \\
&\quad -[\Delta^{\mu\nu}\dot{W}_\nu + W^\mu\theta + (W\cdot\nabla)u^\mu]
\end{aligned}
$$

Here $\sigma^{\mu\nu} \equiv \nabla^{\langle\mu}u^{\nu\rangle}$ is the velocity shear tensor.

Depending on frame, can set either $V^\mu = 0$ or $W^\mu = 0$. In Landau frame ($W^\mu = 0$) and for baryon-free systems ($n = 0$, no heat conduction) equations simplify to:

$$
\begin{aligned}
\dot{e} &= -(e+p+\Pi)\,\theta + \pi_{\mu\nu}\sigma^{\mu\nu} \\
(e{+}p{+}\Pi)\,\dot{u}^\mu &= \nabla^\mu(p+\Pi) - \Delta^{\mu\nu}\nabla^\sigma\pi_{\nu\sigma} + \pi^{\mu\nu}\dot{u}_\nu
\end{aligned}
$$

Need 6 extra equations for bulk and shear viscous pressures Π, $\pi^{\mu\nu}$ $\Longrightarrow$ different paths (Navier-Stokes, Israel-Stewart, Öttinger-Grmela, BRSSS, . . .)

Here we follow Chapman-Enskog strategy: write $f(x,p) = f_{\rm eq}\big(p{\cdot}u(x); T(x), \mu(x)\big) + \delta f(x,p)$ and assume that $\delta f \ll f$ (and thus δN^μ and $\delta T^{\mu\nu}$) can be expanded in gradients of equilibrium parameters T, μ, u_μ.

The second law of thermodynamics (I)

In equilibrium the identity $Ts = p - \mu n + e$ can be rewritten as

$$S^\mu_{\mathrm{eq}} = p(\alpha,\beta)\beta^\mu - \alpha N^\mu_{\mathrm{eq}} + \beta_\nu T^{\mu\nu}_{\mathrm{eq}}$$

where $\alpha \equiv \mu/T$, $\beta \equiv 1/T$, and $\beta^\mu \equiv u^\mu/T$.

The most general off-equilibrium generalization is (Israel & Stewart 1979)

$$S^\mu = p(\alpha,\beta)\beta^\mu - \alpha N^\mu + \beta_\nu T^{\mu\nu} + Q^\mu(\delta N^\mu, \delta T^{\mu\nu})$$

where Q^μ is second and higher order in the off-equilibrium deviations δN^μ and $\delta T^{\mu\nu}$.

The Gibbs-Duhem relation $dp = s\,dT + n\,d\mu$ can be recast as

$$\partial_\mu(p(\alpha,\beta)\beta^\mu) = N^\mu_{\mathrm{eq}}\partial_\mu\alpha - T^{\mu\nu}_{\mathrm{eq}}\partial_\mu\beta_\nu$$

Using also the conservation laws, the entropy production rate takes the form

$$\partial_\mu S^\mu = -\delta N^\mu \partial_\mu\alpha + \delta T^{\mu\nu}\partial_\mu\beta_\nu + \partial_\mu Q^\mu$$

The second law of thermodynamics (II)

In the Chapman-Enskog spirit, one now postulates linear relations between the off-equilibrium flows δN^μ, $\delta T^{\mu\nu}$ and the thermodynamic forces $\partial^\mu \alpha$, $\partial^{(\mu}\beta^{\nu)}$, consistent with the second law

$$\partial_\mu S^\mu = -\delta N^\mu \partial_\mu \alpha + \delta T^{\mu\nu} \partial_\mu \beta_\nu + \partial_\mu Q^\mu \geq 0$$

These relations depend on the choice of Q^μ. Standard dissipative relativistic fluid dynamics assumes $Q^\mu = 0$. In this case

$$T\partial_\mu S^\mu = \Pi X - q^\mu X_\mu + \pi^{\mu\nu} X_{\langle\mu\nu\rangle} \equiv \frac{\Pi^2}{\zeta} - \frac{q^\mu q_\mu}{2\lambda T} + \frac{\pi^{\alpha\beta}\pi_{\alpha\beta}}{2\eta} \geq 0,$$

with thermodynamic forces $X \equiv -\nabla \cdot u = -\theta$, $X^\mu \equiv \frac{\nabla^\mu T}{T} - \dot{u}^\mu = -\frac{nT}{e+p} \nabla^\mu \left(\frac{\mu}{T}\right)$ and $X_{\langle\mu\nu\rangle} \equiv \sigma_{\mu\nu} =$ can be satisfied by setting

$$\Pi = -\zeta\theta, \quad q^\mu = -\lambda \frac{nT^2}{e+p} \nabla^\mu \left(\frac{\mu}{T}\right), \quad \pi^{\mu\nu} = 2\eta\sigma^{\mu\nu}$$

with positive transport coefficents $\zeta \geq 0$, $\lambda \geq 0$, and $\eta \geq 0$ (relativistic Navier-Stokes theory).
Unfortunately, plugging these equations for Π, q^μ, and $\pi^{\mu\nu}$ directly into the non-ideal hydro equations leads to acausal signal propagation.

The second law of thermodynamics (III)

Causal relativistic fluid dynamics requires keeping Q^μ in the entropy flux, at least up to terms of second order in the irreversible flows.

$$S^\mu = s u^\mu + \frac{q^\mu}{T} + Q^\mu$$

Setting $q^\nu = 0$ ($n = 0$) for simplicity, we get up to second order

$$S^\mu = s u^\mu - (\beta_0 \Pi^2 + \beta_2 \pi_{\nu\lambda}\pi^{\nu\lambda})\frac{u^\mu}{2T}$$

This yields (after some algebra)

$$\begin{aligned} T\partial_\mu S^\mu &= \Pi\left[-\theta - \beta_0\dot{\Pi} - \Pi\, T\partial_\mu\left(\frac{\beta_0 u^\mu}{2T}\right)\right] \\ &+ \pi^{\alpha\beta}\left[\sigma_{\alpha\beta} - \beta_2\dot{\pi}_{\alpha\beta} - \pi_{\alpha\beta}\, T\partial_\mu\left(\frac{\beta_2 u^\mu}{2T}\right)\right] \\ &\stackrel{!}{=} \frac{\Pi^2}{\zeta} + \frac{\pi^{\alpha\beta}\pi_{\alpha\beta}}{2\eta} \geq 0 \end{aligned}$$

The thermodynamic forces $-\theta$, $\sigma_{\alpha\beta}$ are seen to be self-consistently modified by the irreversible flows Π, $\pi_{\alpha\beta}$. This leads to dynamical ("transport") equations for Π, $\pi_{\alpha\beta}$.

Transport equations for the irreversible flows

The resulting transport equations for Π, $\pi_{\alpha\beta}$ are (Israel & Stewart 1979, Muronga 2002, 2004)

$$
\begin{aligned}
\dot{\Pi} &= -\frac{1}{\tau_\Pi}\left[\Pi + \zeta\theta + \Pi\zeta T\partial_\mu\left(\frac{\tau_\Pi u^\mu}{2\zeta T}\right)\right] = -\frac{1}{\tau'_\Pi}\left[\Pi + \zeta'\theta\right] \\
\Delta_{\alpha\mu}\Delta_{\beta\nu}\dot{\pi}^{\mu\nu} &= -\frac{1}{\tau_\pi}\left[\pi_{\alpha\beta} - 2\eta\sigma_{\alpha\beta} + \pi_{\alpha\beta}\eta T\partial_\mu\left(\frac{\tau_\pi u^\mu}{2\eta T}\right)\right] \\
&\quad + \text{terms that don't generate entropy} \\
&= -\frac{1}{\tau'_\pi}\left[\pi_{\alpha\beta} - 2\eta'\sigma_{\alpha\beta}\right] + \dots
\end{aligned}
$$

Here we introduced the relaxation times $\tau_\Pi = \zeta\beta_0$, $\tau_\pi = 2\eta\beta_2$, and $\tau'_\Pi = \frac{\tau_\Pi}{1+\zeta\gamma_\Pi}$, $\zeta' = \frac{\zeta}{1+\eta\gamma_\Pi}$,

$\tau'_\pi = \frac{\tau_\pi}{1+\eta\gamma_\pi}$, $\eta' = \frac{\eta}{1+\eta\gamma_\pi}$, where $\gamma_\Pi \equiv T\partial_\mu\left(\frac{\tau_\Pi u^\mu}{2\zeta T}\right)$ and $\gamma_\pi \equiv T\partial_\mu\left(\frac{\tau_\pi u^\mu}{2\eta T}\right)$.

(UH, Song, Chaudhuri 2006)

ζ, η and τ_Π, τ_π should be calculated from the underlying microscopic theory. This has been done by KSS and BRSSS for infinitely strongly coupled SYM theory, and by AMY and YM for weakly coupled QCD in Boltzmann transport theory (see below).

The purple terms kick in wherever the expansion rate gets large and then effectively reduce the viscosities and relaxation times. – The viscous pressures $\Pi, \pi^{\mu\nu}$ relax exponentially towards their (flow-modified) Navier-Stokes limits on (flow-modified) microscopic relaxation time scales τ'_Π, τ'_π.

More second-order terms . . .

Analyzing the second law of thermodynamics misses second-order terms that don't contribute to 2nd order entropy production but may still affect the evolution of flow.

For systems with conformal symmetry (Π=0) and vanishing chemical potentials (q^μ=0) BRSSS (Baier, Romatschke, Son, Starinets, Stephanov, JHEP 04 (2006) 100) found 5 possible second-order terms:

$$\boxed{\begin{aligned}\pi^{\mu\nu} = {} & 2\eta\sigma^{\mu\nu} - \tau_\pi\left[\Delta^{\mu\alpha}\Delta^{\nu\beta}\dot{\pi}_{\alpha\beta} + \frac{4}{3}\theta\pi^{\mu\nu}\right] - \frac{\lambda_1}{2\eta^2}\pi^{\langle\mu}{}_\alpha\pi^{\nu\rangle\alpha} - \frac{\lambda_2}{2\eta}\pi^{\langle\mu}{}_\alpha\omega^{\nu\rangle\alpha} \\ & -\frac{\lambda_3}{2}\omega^{\langle\mu}{}_\alpha\omega^{\nu\rangle\alpha} + \frac{\kappa}{2}\left[R^{\langle\mu\nu\rangle} + 2u_\alpha R^{\alpha\langle\mu\nu\rangle\beta}u_\beta\right]\end{aligned}}$$

where $\omega_{\mu\nu} = \frac{1}{2}(\nabla_\mu u_\nu - \nabla_\nu u_\mu)$ is the vorticity and $R^{\alpha\mu\nu\beta}$, $R^{\mu\nu}$ are the Riemann and Ricci tensors, respectively.

Now we have **5** second order coefficients τ_π, λ_1, λ_2, λ_3, κ, in addition to η.

Betz, Henkel and Rischke (arXiv:0812.1440 [nucl-th]) generalized this to include heat conduction and bulk viscosity $\Longrightarrow$ even more coefficients . . .

Weak and strong coupling limits of shear viscosity and second order coefficients

(zero masses and chemical potentials)

Ratio	pQCD ($N_f = 3$) (AMY '00,'03; YM '08)	SYM (PSS '01; KSS '05; BRSSS '08)
$\frac{\eta}{s}$	$\frac{46.1}{N_c^2 g^4 \ln(4.17/g\sqrt{N_c})} \approx 1.7 \ (g=2)$	$\frac{1}{4\pi} \approx 0.08$
$\frac{(e+p)\tau_\pi}{\eta}$	5 to 5.9	$4 - 2\ln 2 \approx 2.6137$
$\frac{(e+p)\lambda_1}{\eta^2}$	5.2 to 4.1	2
$\frac{(e+p)\lambda_2}{\eta^2}$	$-2\eta\frac{(e+p)\tau_\pi}{\eta} = -10$ to -11.8	$-4\ln 2 \approx -2.7726$
$\frac{(e+p)\lambda_3}{\eta^2}$	0	0
$\frac{(e+p)\kappa}{\eta^2}$	0	4

AMY = Arnold/Moore/Yaffe; YM = York & Moore; PSS = Policastro/Son/Starinets; KSS = Kovtun/Son/Starinets; BRSSS = Baier/Romatschke/Son/Starinets/Stephanov

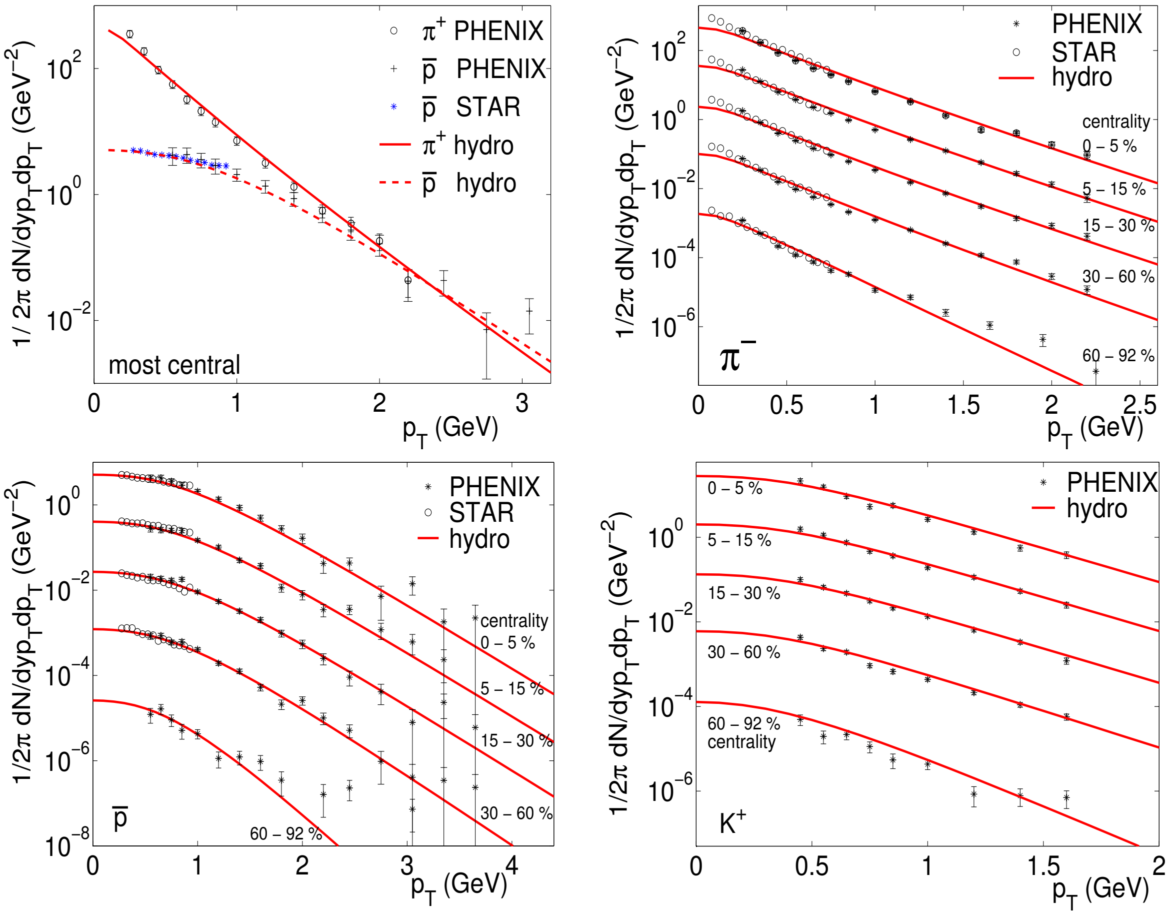
Fortunately, terms $\sim \lambda_{1,2,3}$ appear to be numerically unimportant for hydro evolution.

1. **Formalism**

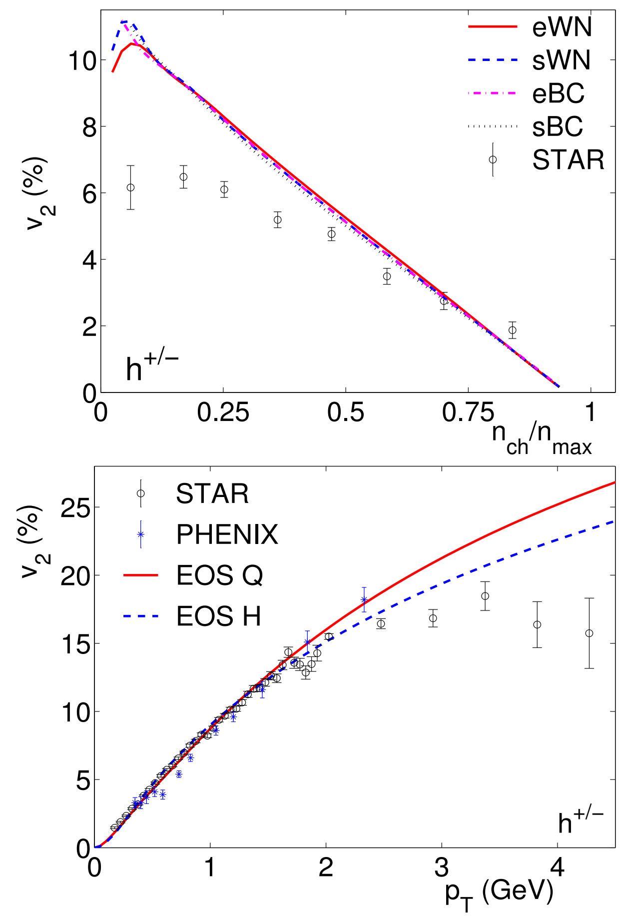
2. **Numerical implementation and results**

Successes of hydrodynamics at RHIC:

Single particle spectra from central and peripheral Au+Au @ 130 A GeV (STAR, PHENIX):



Centrality and momentum dependence of elliptic flow v_2 (STAR, PHENIX, PHOBOS):



Model parameters fixed with π, $\bar{p}$ spectra at $b = 0$; all other spectra predicted (UH & P.Kolb, hep-ph/0204061).

$$v_2 = \langle \cos(2\phi) \rangle$$

What is fitted, what is predicted?

Au+Au @ 130 A GeV:

$\tau_{\rm eq} = 0.6\,\text{fm}/c$, $\quad e_{\max}(b{=}0) = 24.6\,\text{GeV/fm}^3$, $\quad \langle e\rangle(\tau{=}1\,\text{fm}/c) = 5.4\,\text{GeV/fm}^3$

$T_{\max}(b{=}0) = 340\,\text{MeV}$, $\quad T_{\rm chem} = T_{\rm had} = 165\,\text{MeV}$, $\quad T_{\rm dec} = 130\,\text{MeV}$

All fit parameters are fixed in central ($b{=}0$) collisions:

- Glauber model $\Rightarrow$ shape of initial transverse entropy and baryon density profiles $s(\boldsymbol{r},\tau_{\rm eq}), n_B(\boldsymbol{r},\tau_{\rm eq})$
  $\Rightarrow$ free parameters $s_0(\tau_{\rm eq})$, $n_0(\tau_{\rm eq})$, soft/hard fraction
- Measured p/π ratio $\Rightarrow$ fixes n_0/s_0
- Total charged multiplicity $dN_{\rm ch}/dy \Rightarrow$ fixes product $\tau_{\rm eq}\cdot s_0(\tau_{\rm eq})$
- soft/hard fraction $\Rightarrow$ fixed through centrality dependence of $dN_{\rm ch}/dy$
- Shape of π, p spectra $\Rightarrow$ fixes decoupling temperature $T_{\rm dec}$ and radial flow $\langle v_\perp\rangle$
- Final radial flow $\langle v_\perp\rangle \Rightarrow$ "fixes" $\tau_{\rm eq}$ [upper limit] (flow needs time and pressure to develop)
- Equation of State $\Rightarrow$ compute $e_0 = e_{\max}(b{=}0)$, $T_{\max}(b{=}0)$ from s_0, n_0

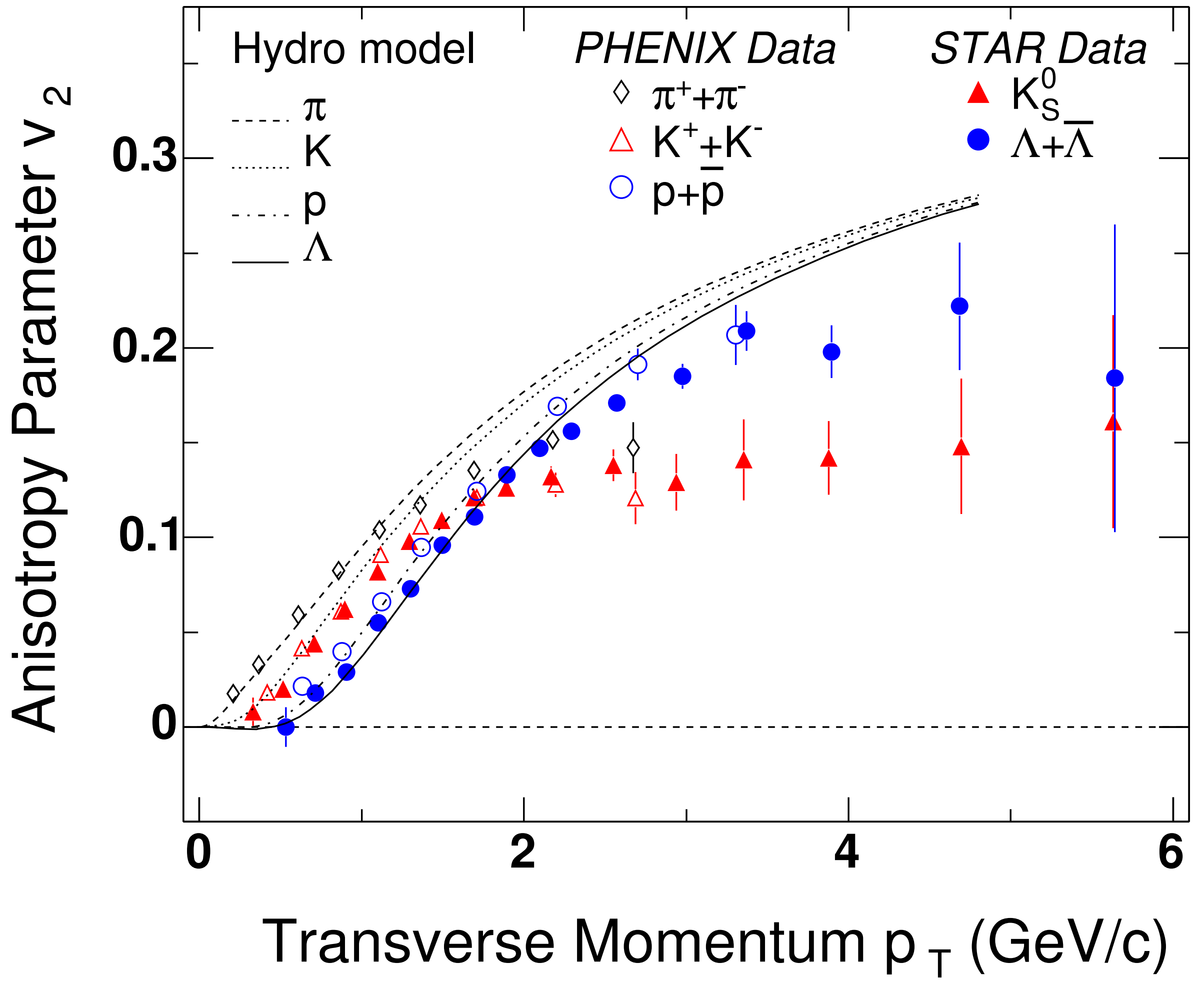
Predictions (no additional parameters!):

- All hadron spectra other than p, π in $b{=}0$ collisions
- All hadron spectra and elliptic flow coefficients for non-central collisions at any impact parameter

Final radial flow $\langle v_\perp\rangle > 0.5\,c \Longrightarrow$ bang!

Perfect fluidity: magnitude and mass dependence of elliptic flow

STAR Coll., PRL 87, 182301 (2001) and PRL 92, 052302 (2004); PHENIX Coll., PRL 91, 182301 (2003)

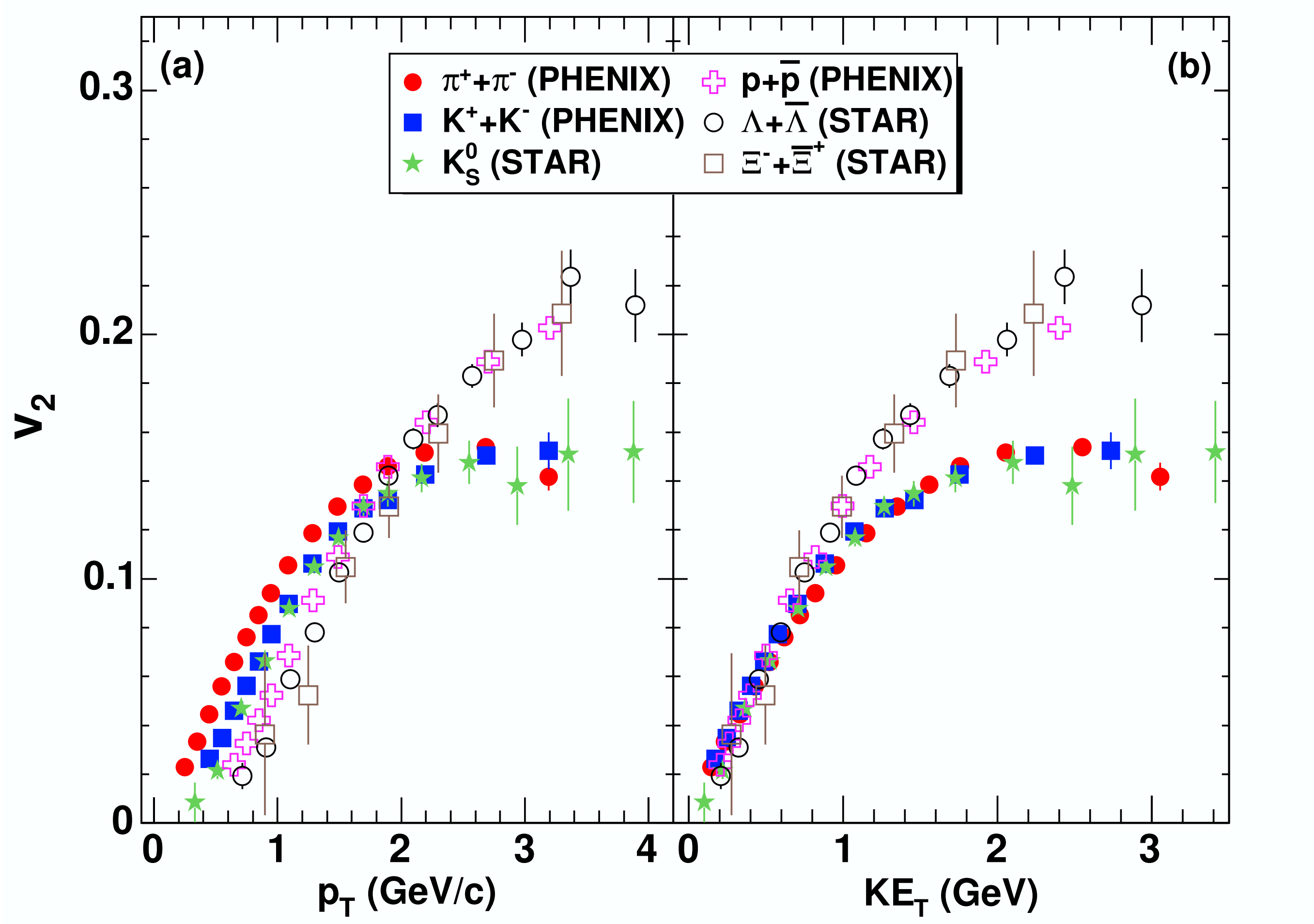


Data follow the hydrodynamically predicted rest mass dependence of $v_2(p_\perp)$ out to $p_\perp \sim 1.5\,\mathrm{GeV}$ for mesons and out to $p_\perp \sim 2.3\,\mathrm{GeV}$ for baryons

$\Longrightarrow$ **bulk of matter ($> 99\%$ of all particles) behaves hydrodynamically!**

Perfect fluidity: m_T-scaling of elliptic flow at low p_T

PHENIX Coll., PRL 98, 162301 (2007)



$$\mathrm{KE_T} \equiv m_T - m_0 = \text{transverse kinetic energy}$$

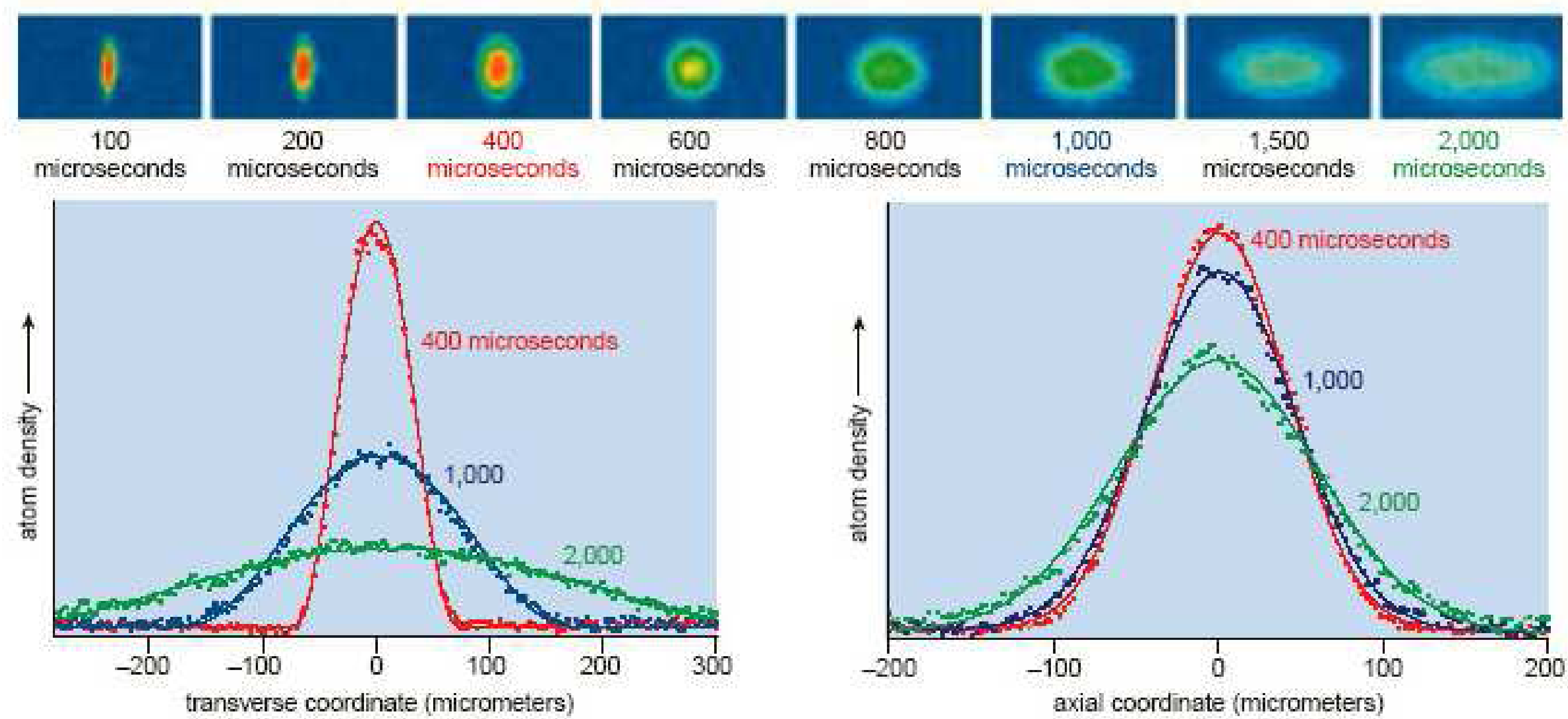
m_T-scaling = evidence for thermalization (almost too good . . . !)

so hydro works –

why? what does this mean?

Elliptic collective flow of strongly coupled atoms at $T = 10^{-6}$ K:

J.E.Thomas et al., Am. Scientist 92 (2004) 238



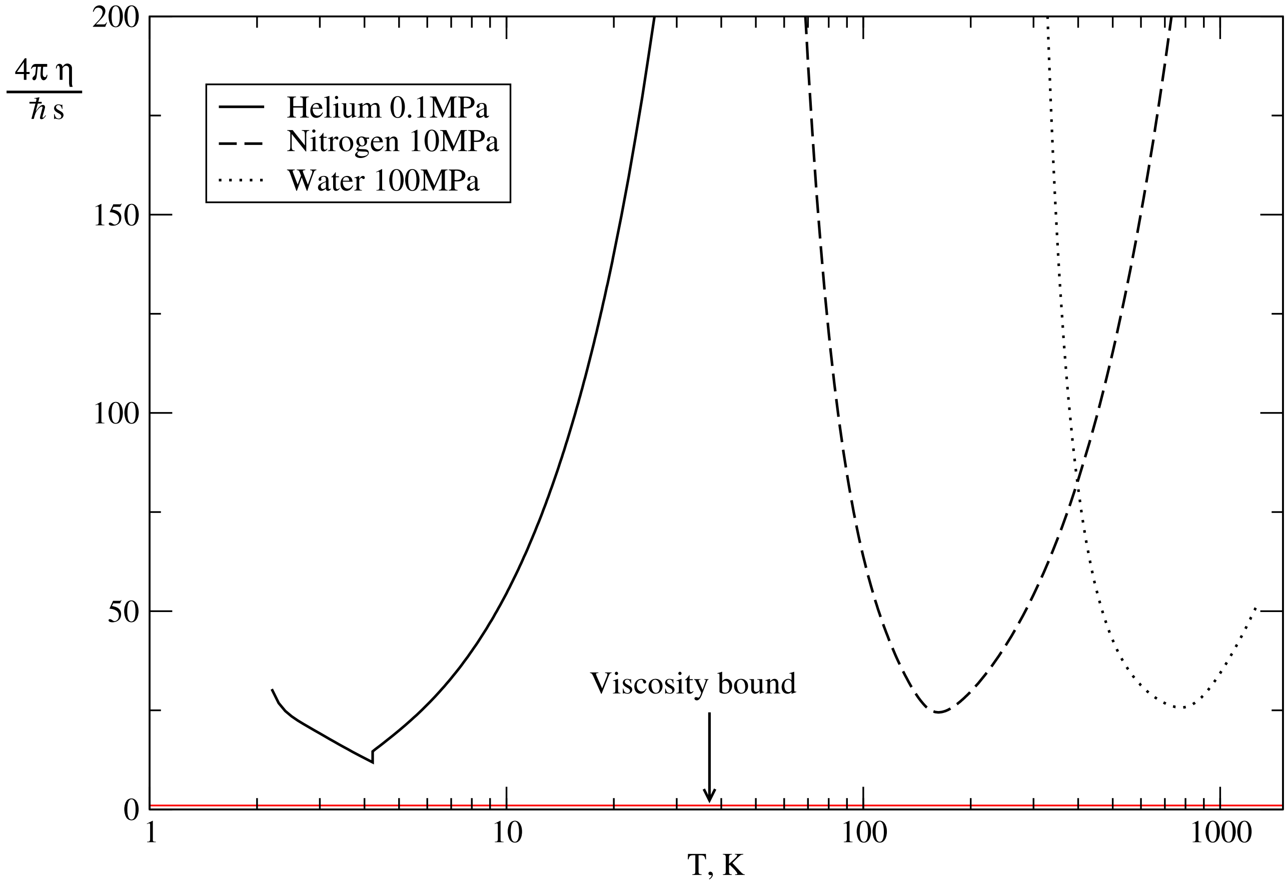
Interaction strength can be tuned (Feshbach resonance):
Strong interaction: elliptic collective flow
Weak interaction: ballistic expansion with aspect ratio $\rightarrow 1$

Success of ideal fluid dynamics requires QGP to be strongly coupled and almost inviscid!

QGP – the most perfect fluid ever observed?

AdS/CFT universal lower viscosity bound conjecture: $\frac{\eta}{s} \geq \frac{\hbar}{4\pi}$

Kovtun, Son, Starinets, PRL 94 (2005) 111601



Upper limit for QGP viscosity from various recent estimates are close to this bound!

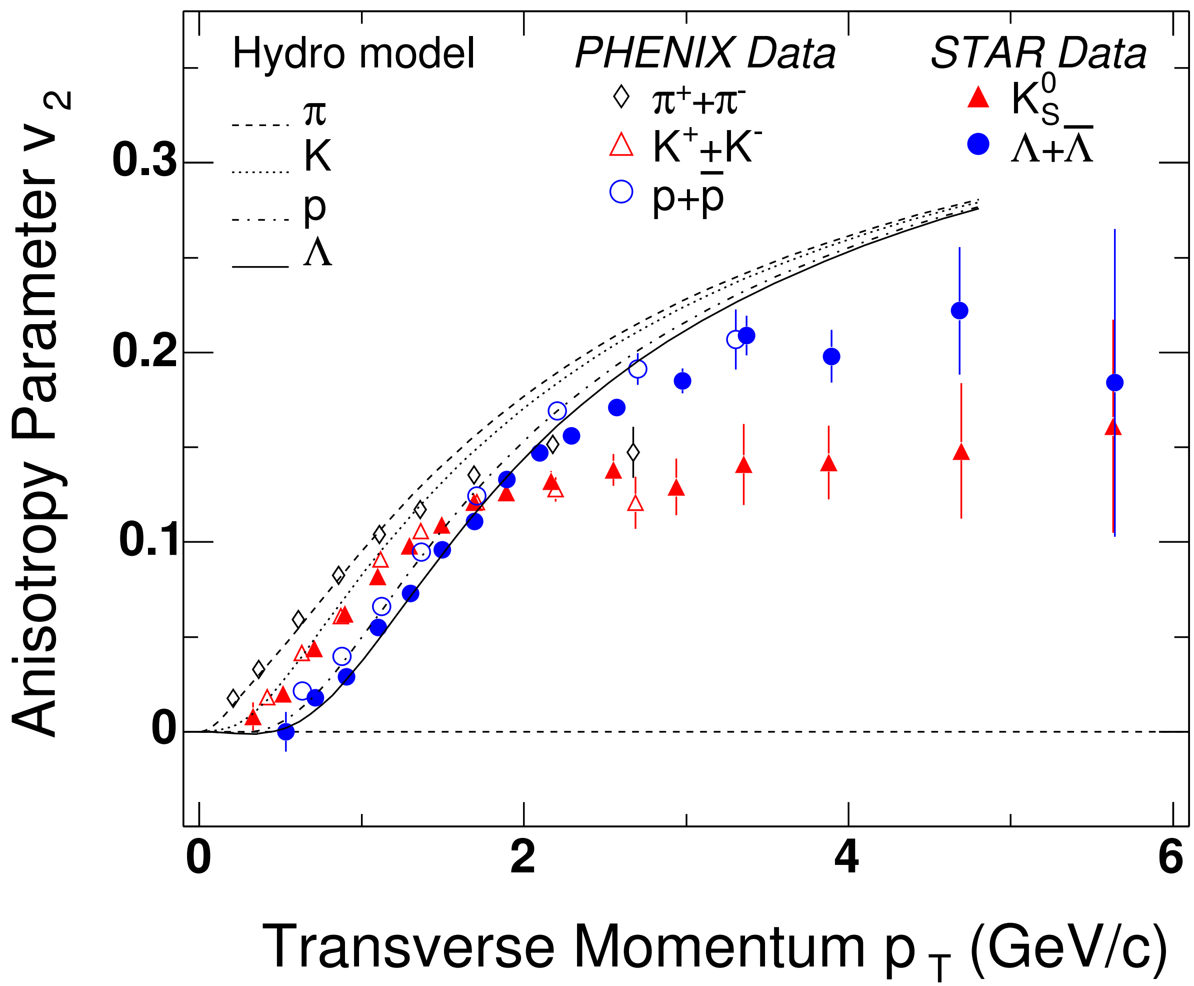
But: quantitative constraint on η/s requires viscous hydrodynamics code.

So ideal hydro works quite well at RHIC.

But there are also indications of non-zero viscosity:

Ideal fluid dynamics breaks down at $p_T \gtrsim 1.5 - 2\,\mathrm{GeV}/c$:

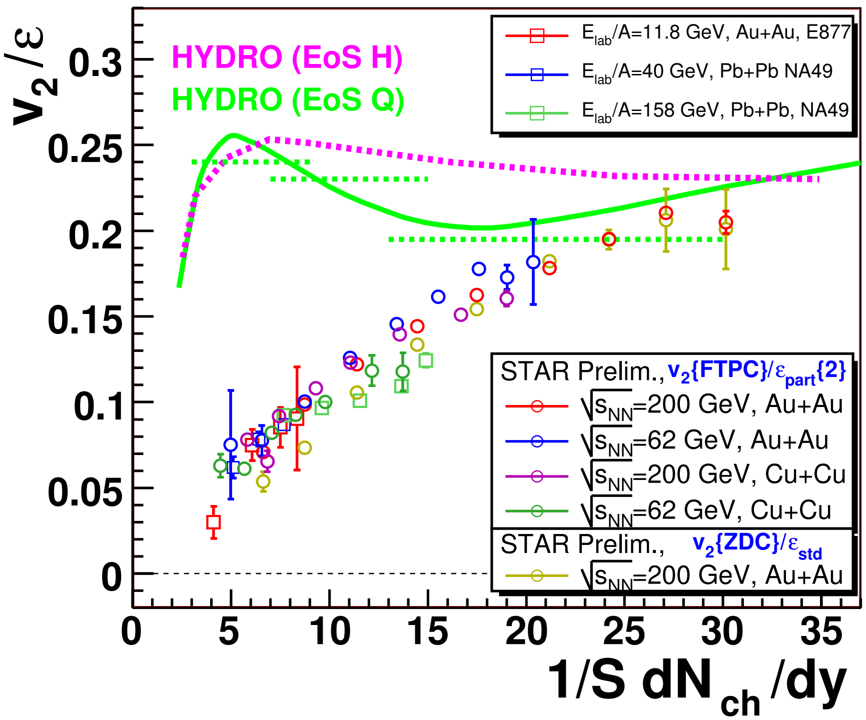
STAR Coll., PRL 87, 182301 (2001) and PRL 92, 052302 (2004); PHENIX Coll., PRL 91, 182301 (2003)



- Consistent with viscous effects during early QGP stage (viscous corrections increase $\sim p_T^2$)
- Can be used to constrain QGP viscosity $\Longrightarrow$ viscous hydrodynamics

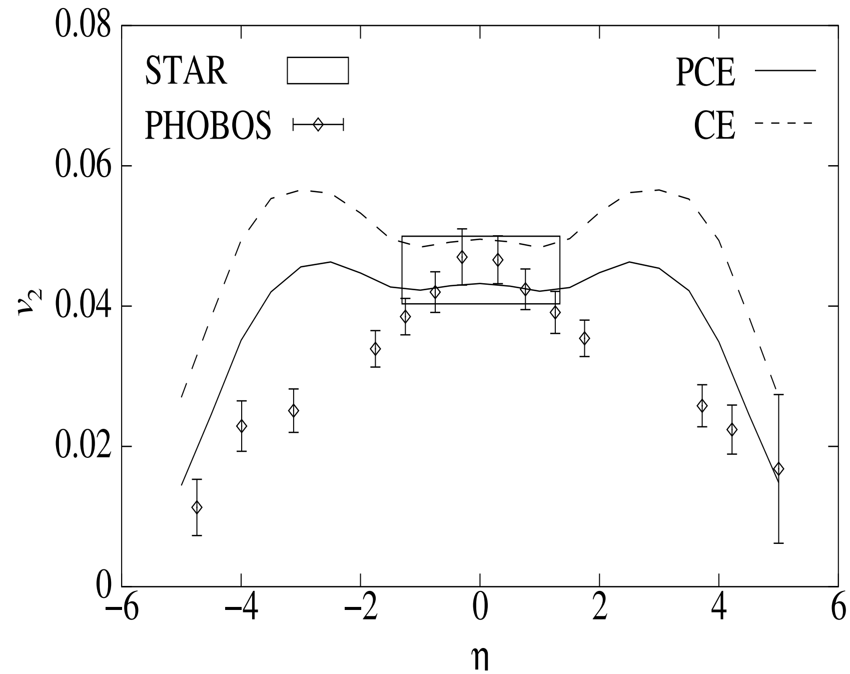
Smaller, less dense collision systems: late hadronic viscosity

S. Voloshin [STAR], JPG 34 (2007) S883



(3+1)-d hydrodynamics:

T. Hirano, PRC 65 ('02) 011901; 66 ('02) 054905



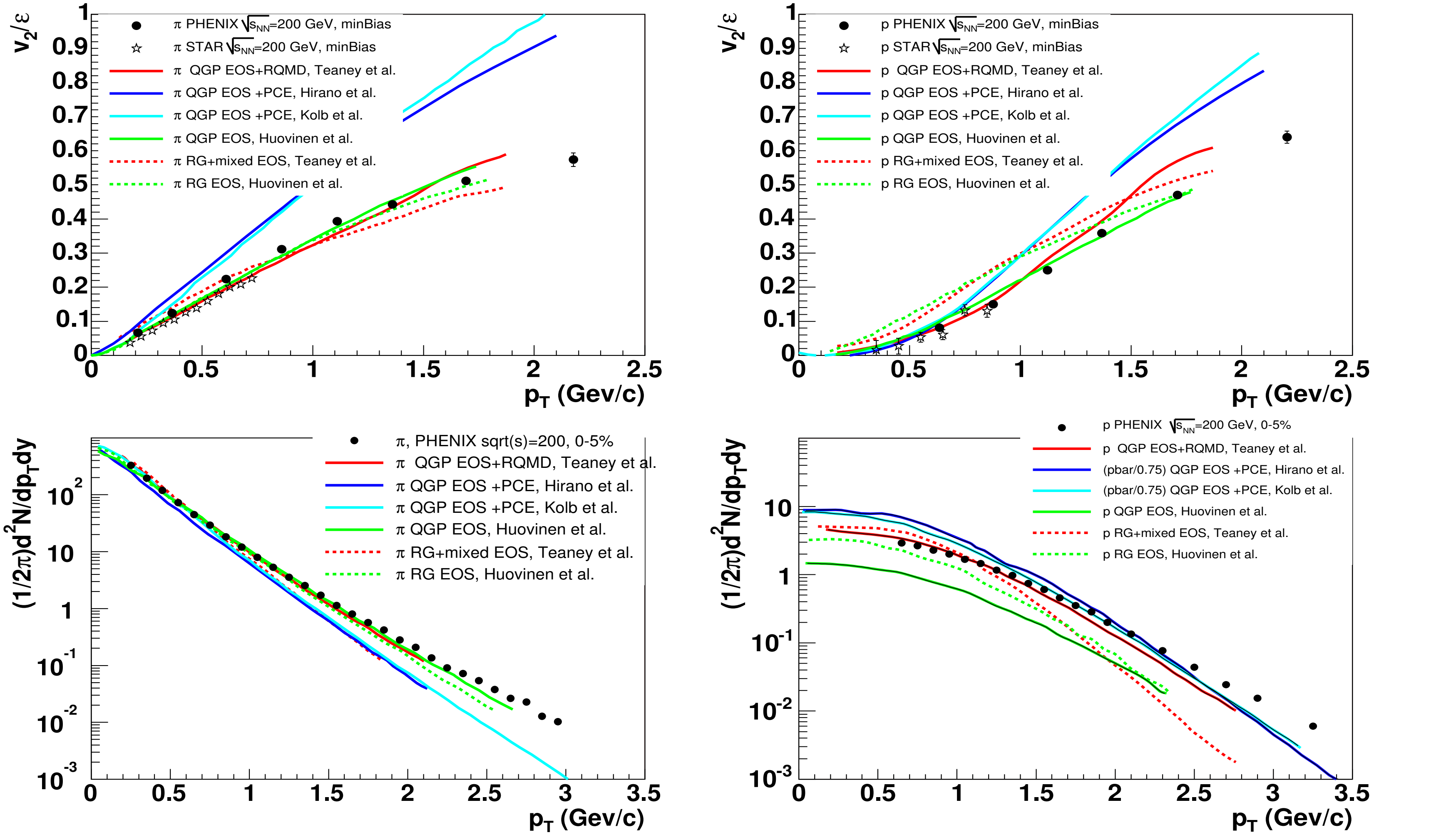
- viscous correction to ideal hydro $\frac{v_2^{\rm measured}}{v_2^{\rm hydro}}$ scales with $\frac{1}{S}\frac{dN_{\rm ch}}{dy} \propto s_{\rm init}$ ("multiplicity scaling")
- ideal hydro limit only approached for $e_{\rm init} > 10\,\mathrm{GeV/fm}^3$

Why? Late hadronic viscosity! (Teaney, Shuryak 2001)

At RHIC, hadronic viscosity & chemical non-equilibrium matter:

PHENIX White Paper, NPA 757 (2005) 184

pions protons



All theory curves use the same hydrodynamics and EOS in QGP phase!

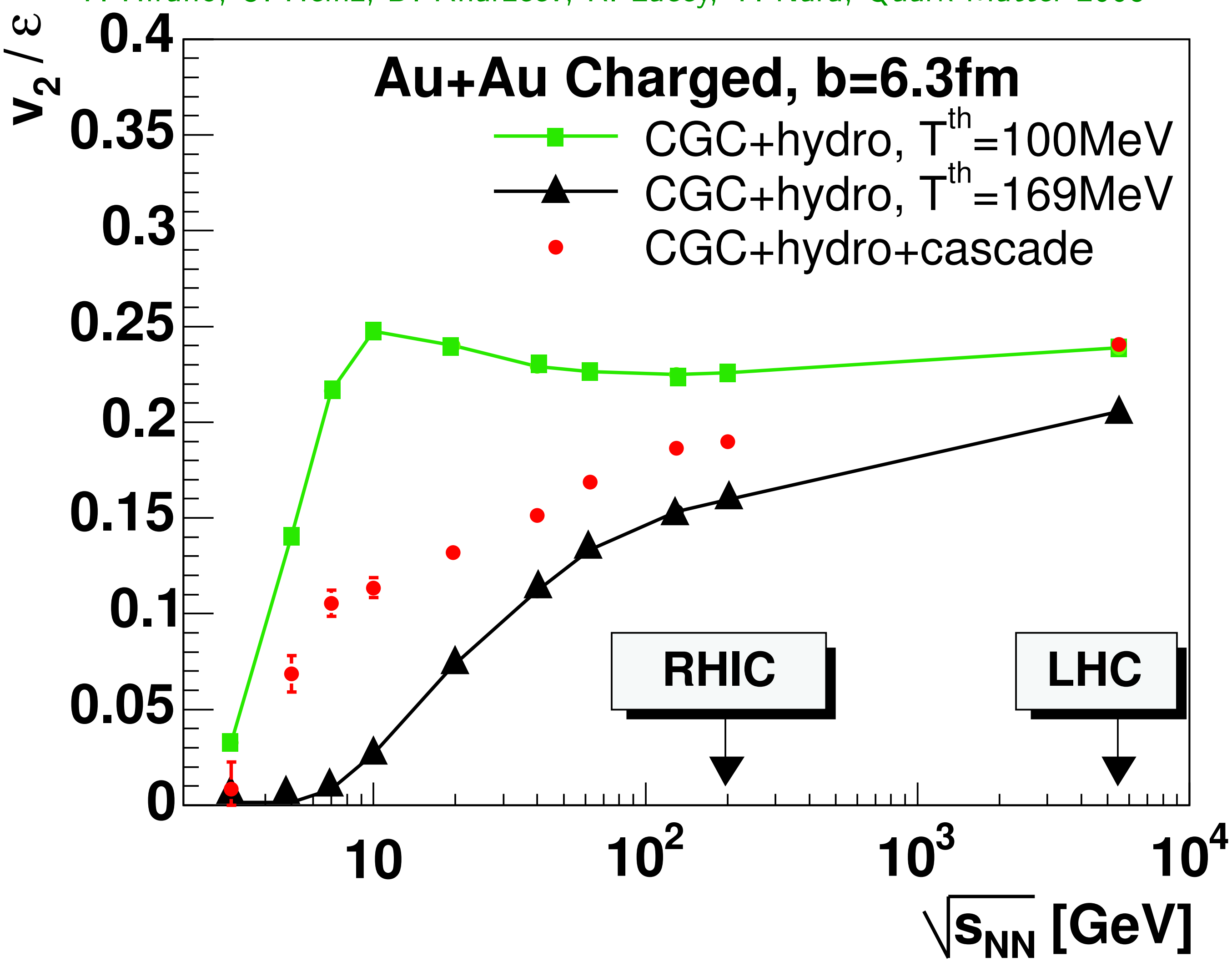
How we deal with the hadron phase makes all the difference . . .

The only model that simultaneously fits all data is hydro+RQMD

(**Teaney & Shuryak 2001**) (see also Hirano et al. 2006 (3D hydro + JAM))

Hadronic dissipation effects disappear at the LHC:

T. Hirano, U. Heinz, D. Kharzeev, R. Lacey, Y. Nara, Quark Matter 2008



At LHC, all momentum anisotropy is created in QGP phase

$\Longrightarrow$ hadronic dissipation effects become negligible

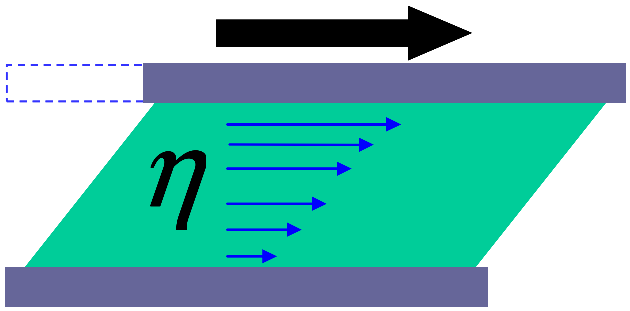
Late hadronic evolution still important for final distribution of momentum anisotropy over particle species (e.g. pions vs. protons)

Relativistic hydrodynamics

for viscous fluids

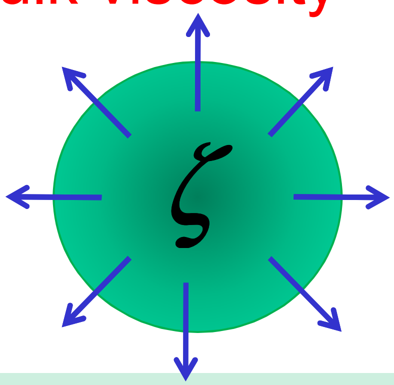
What is viscosity?

Shear viscosity – measures the resistance to flow gradients



acts against the buildup of
flow anisotropy

Bulk viscosity – measures the resistance to expansion



acts against the buildup of
radial flow

Heat conductivity – measures the ability of heat transfer

λ

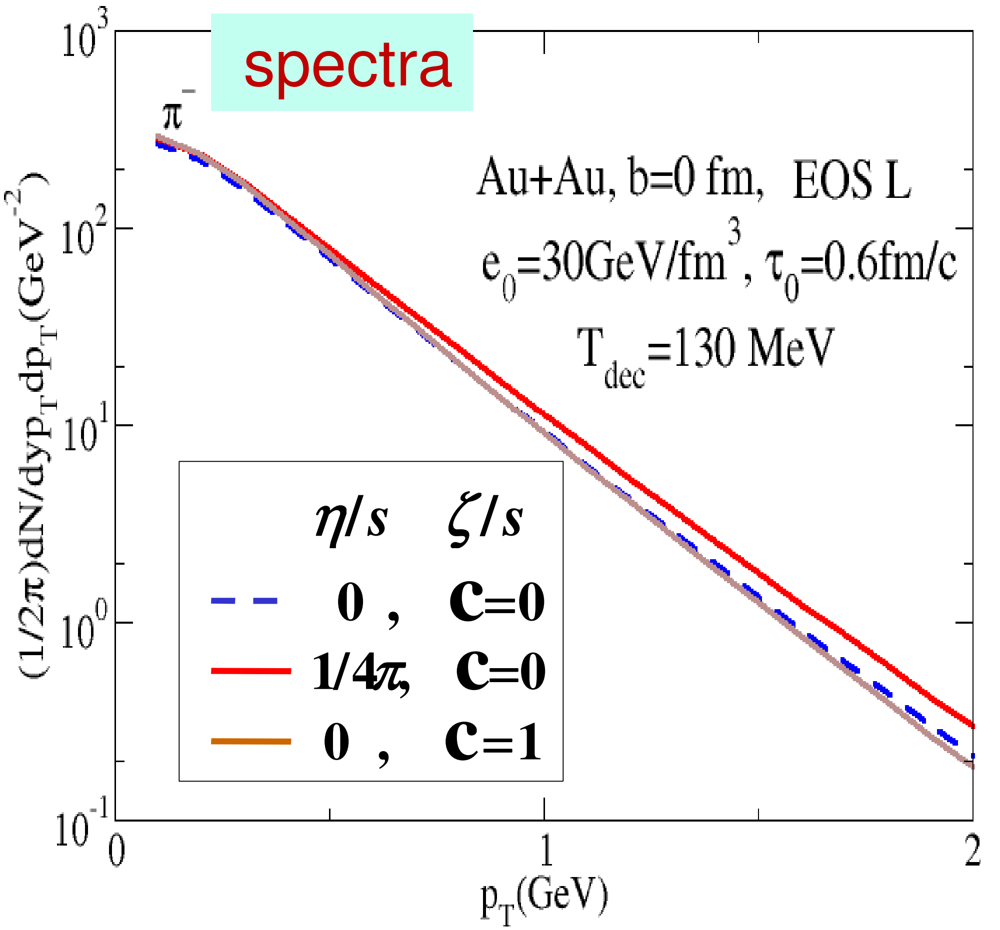
Assume: $\lambda = 0, \quad n \approx 0$ (RHIC&LHC)

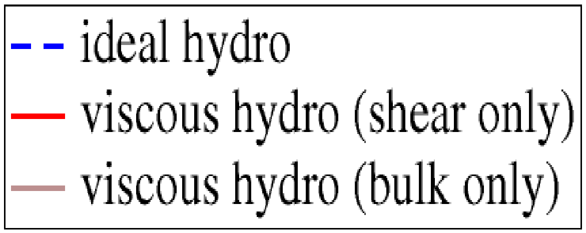
Qualitative effects of η & ζ on spectra

The same initial & final conditions:

- - - ideal hydro —— viscous hydro-shear only ——viscous hydro-bulk only

Song & Heinz, 0909





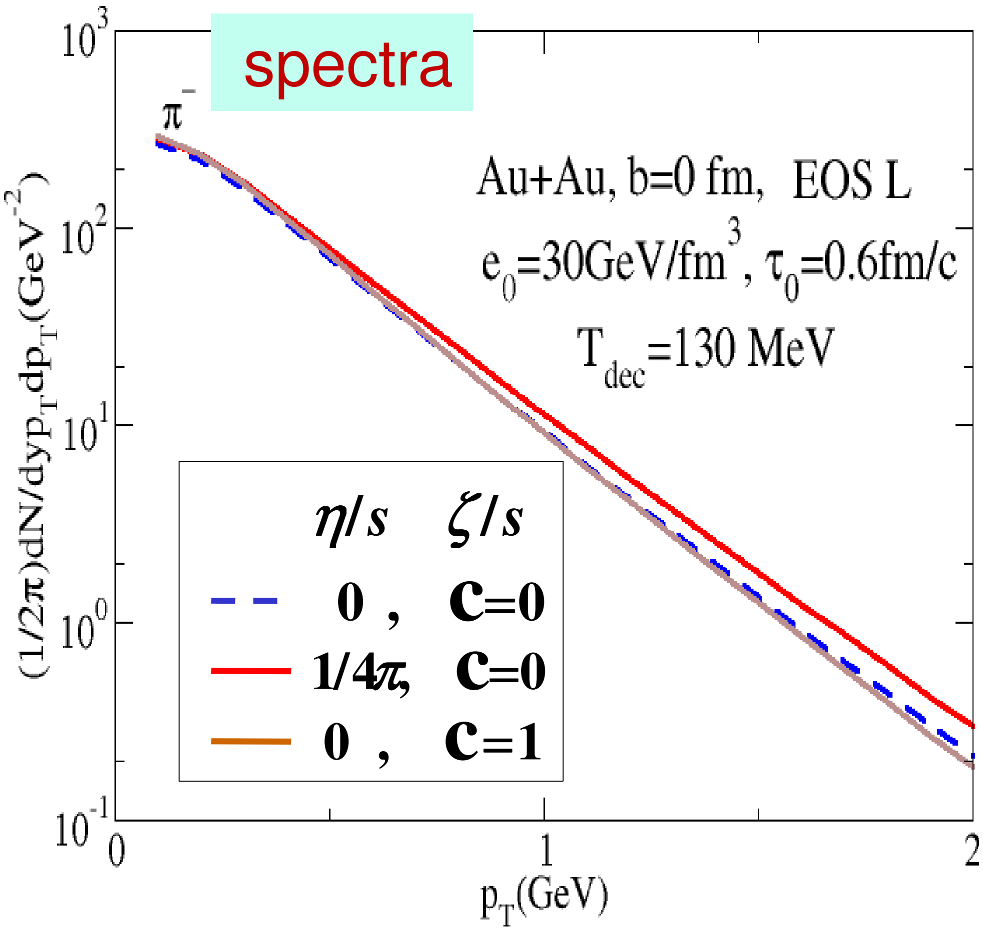
-shear viscosity: flatter spectra

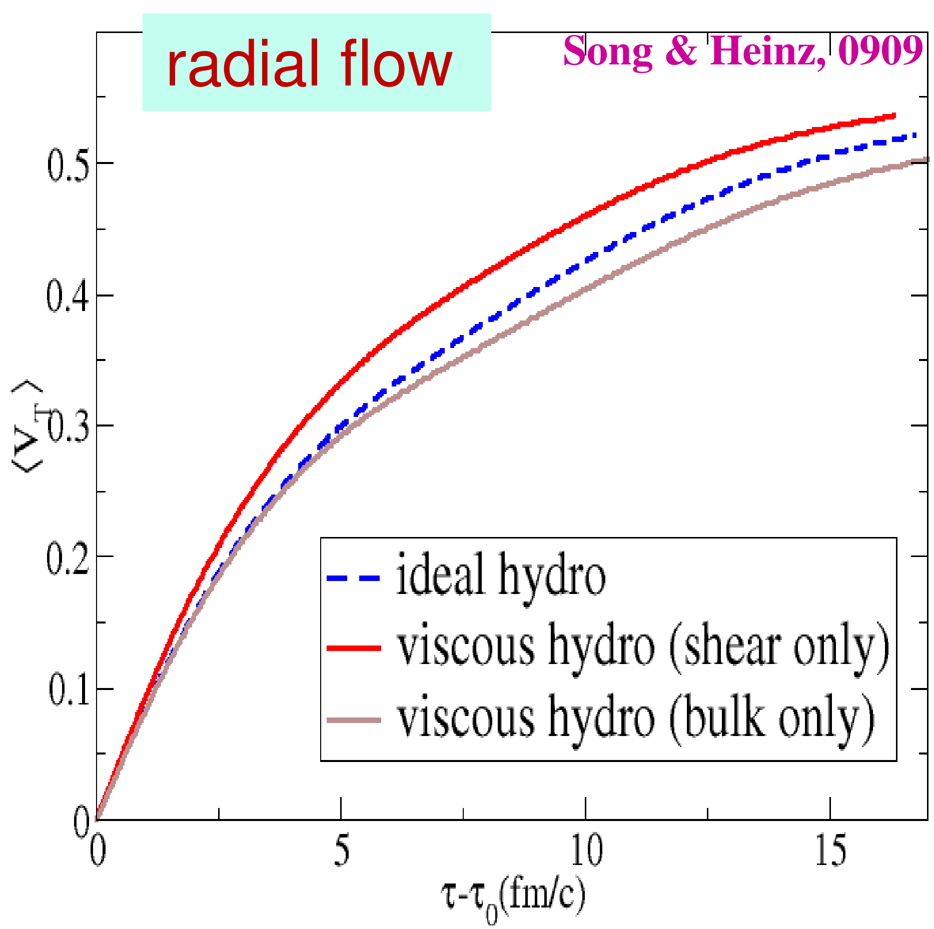
-bulk viscosity: steeper spectra

Qualitative effects of η & ζ on spectra

The same initial & final conditions:

- - - ideal hydro **—— viscous hydro-shear only** **——viscous hydro-bulk only**

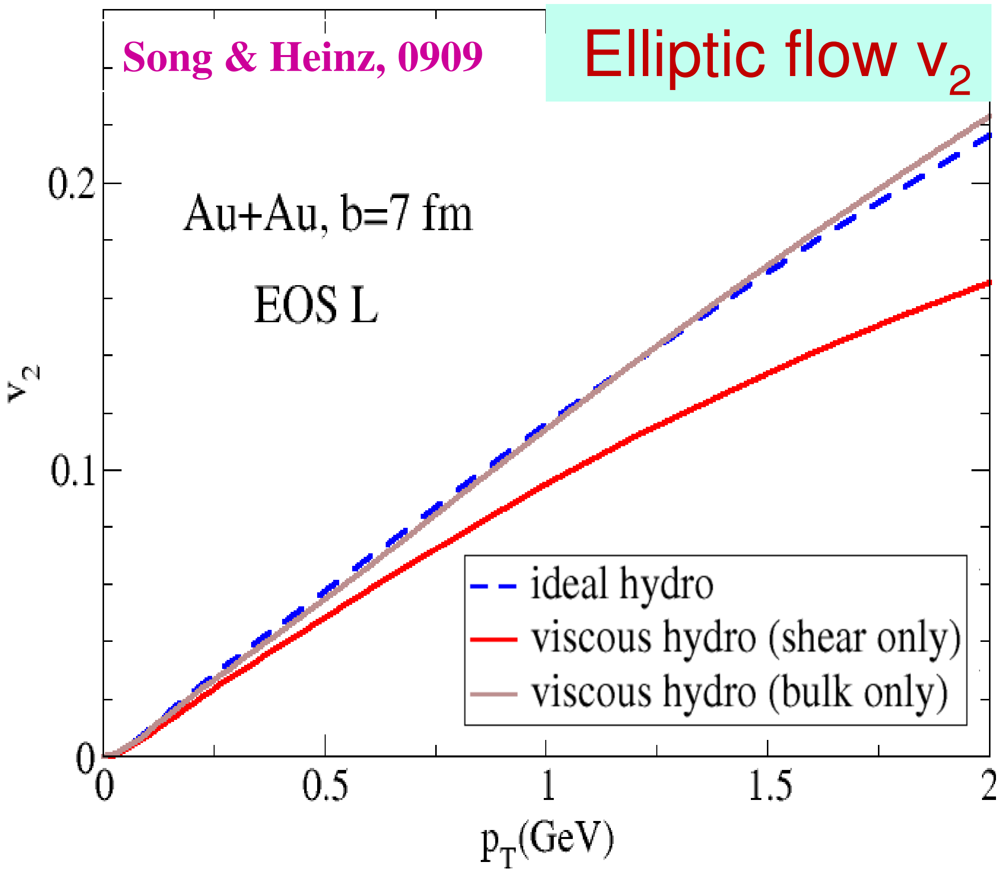




-shear viscosity: flatter spectra; increases radial flow

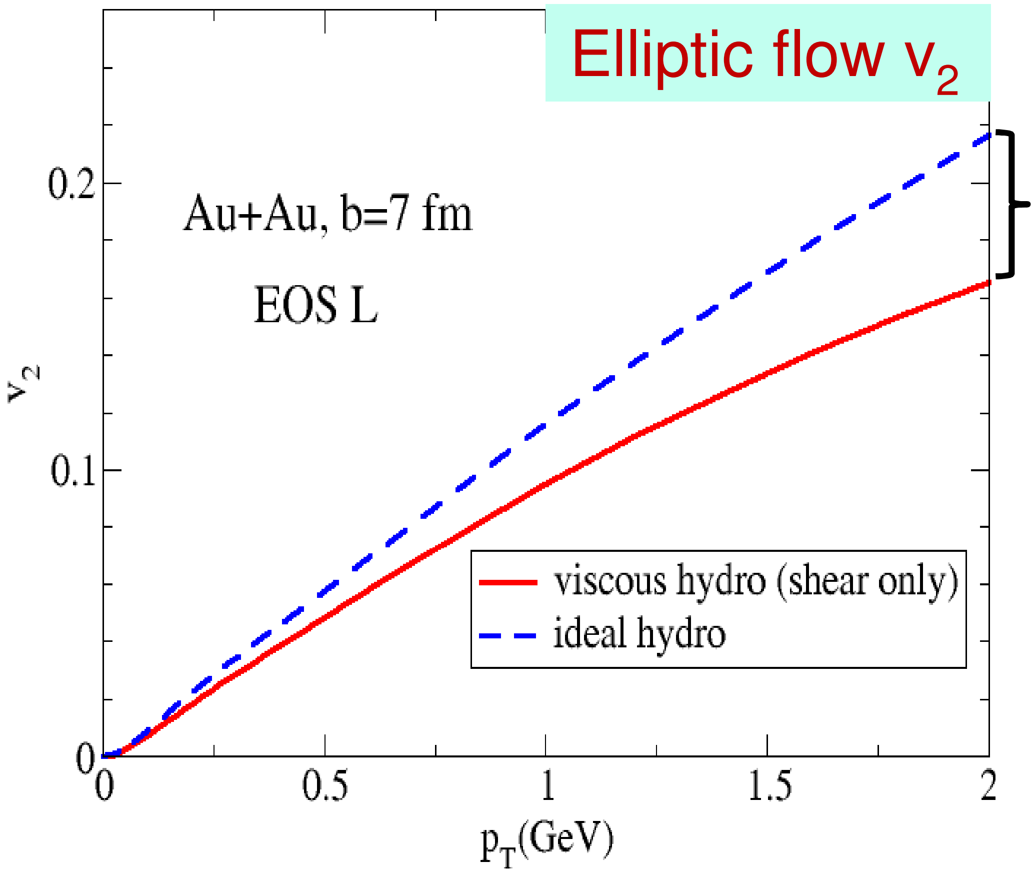
-bulk viscosity: steeper spectra; decreases radial flow

Qualitative effects of η & ζ on v_2



-both **shear** & **bulk** viscosity suppress $\mathbf{v_2}$ at low $\mathbf{p_T}$

Shear viscosity η & elliptic flow v_2



20-25% v_2 suppression $\frac{\eta}{s} = \frac{1}{4\pi}$

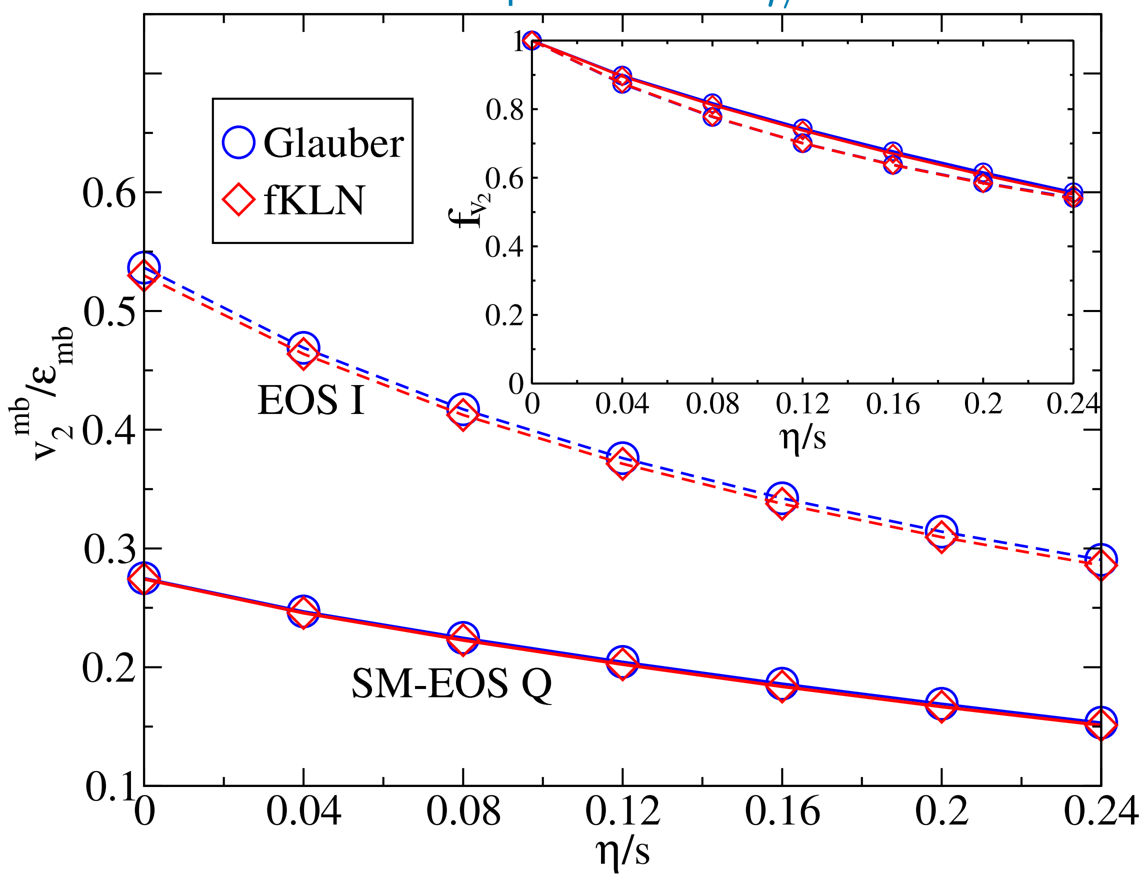
- **v_2** can be used to extract the QGP shear viscosity

- for an accurate extraction of QGP viscosity, one needs very precise **v_2** (experimental data & theoretical results)

Viscous suppression of eccentricity-scaled elliptic flow v_2/ε

(UH, Moreland, Song, arXiv:0908.2617)

Scaled elliptic flow vs. η/s



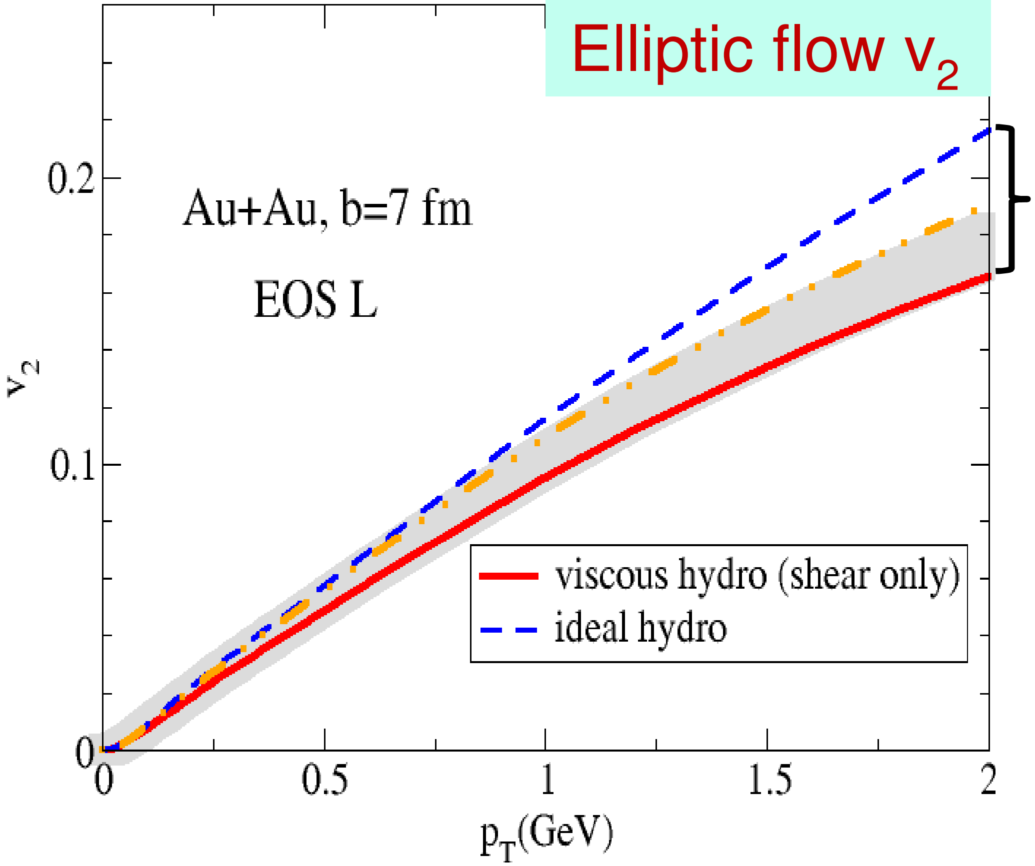
Shear viscosity η/s suppresses elliptic flow:

- $f_{v_2} = \frac{\left(v_2^{\rm mb}/\varepsilon_{\rm mb}\right)_{\rm viscous}}{\left(v_2^{\rm mb}/\varepsilon_{\rm mb}\right)_{\rm ideal}}$ (mb = min.bias)
- initial source eccentricity $\varepsilon = \frac{\langle\langle y^2-x^2\rangle\rangle}{\langle\langle y^2+x^2\rangle\rangle}$
- fKLN model: H.-J. Drescher et al., PRC 74 (2006) 044905
- curves calculated with `VISH2+1` – viscous Israel-Stewart hydro with longitudinal boost-invariance (H. Song 2007-2008)

- Even small $\eta/s \sim 1/4\pi$ leads to sizeable $(\sim 20\%)$ suppression of elliptic flow $\Longrightarrow$ easily measurable if ideal fluid baseline is known
- Viscous suppression of v_2/ε relative to ideal hydro is a unique function of η/s, independent of initial source eccentricity and $\approx$ independent of EOS.
- **But:** 15% uncertainty in initial source eccentricity up to $\Longrightarrow$ $\mathcal{O}(100\%)$ uncertainty for η/s!

Shear viscosity η & elliptic flow v_2

Elliptic flow v_2



20-25% v_2 suppression $\frac{\eta}{s} = \frac{1}{4\pi}$

- v_2 can be used to extract the QGP shear viscosity

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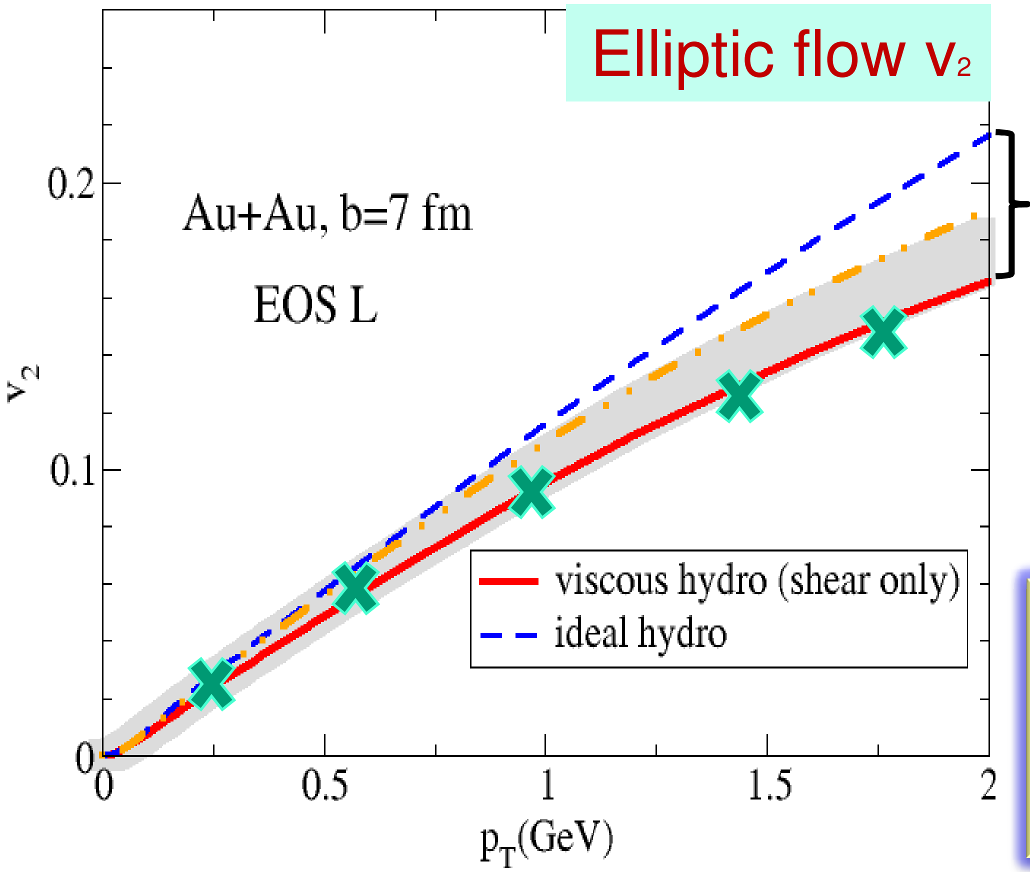
Shear viscosity η & elliptic flow v_2



20-25% v_2 suppression $\quad \frac{\eta}{s} = \frac{1}{4\pi}$

Example: If v_2 is increased by 10%, one needs to increase η / s by $\mathbf{0.5/(4\pi)}$ in order to describe the same exp. data

for $\eta / s \sim O(1/4\pi)$
-10% uncertainties in v_2 translate into **50%** uncertainties for the extracted value of η / s

- v_2 can be used to extract the QGP shear viscosity
- for an accurate extraction of QGP viscosity, one needs very precise v_2 (experimental data & theoretical results)

Extracting η/s from v_2 data

Input / parameters for viscous hydro

- **initial conditions:** $\tau_0, e_0(x, y, b) / s_0(x, y, b)$
  - **initial eccentricity** (Glauber vs. CGC ; optical vs. fluctuating)

- **treatment of hadronic stage and freeze-out procedure**

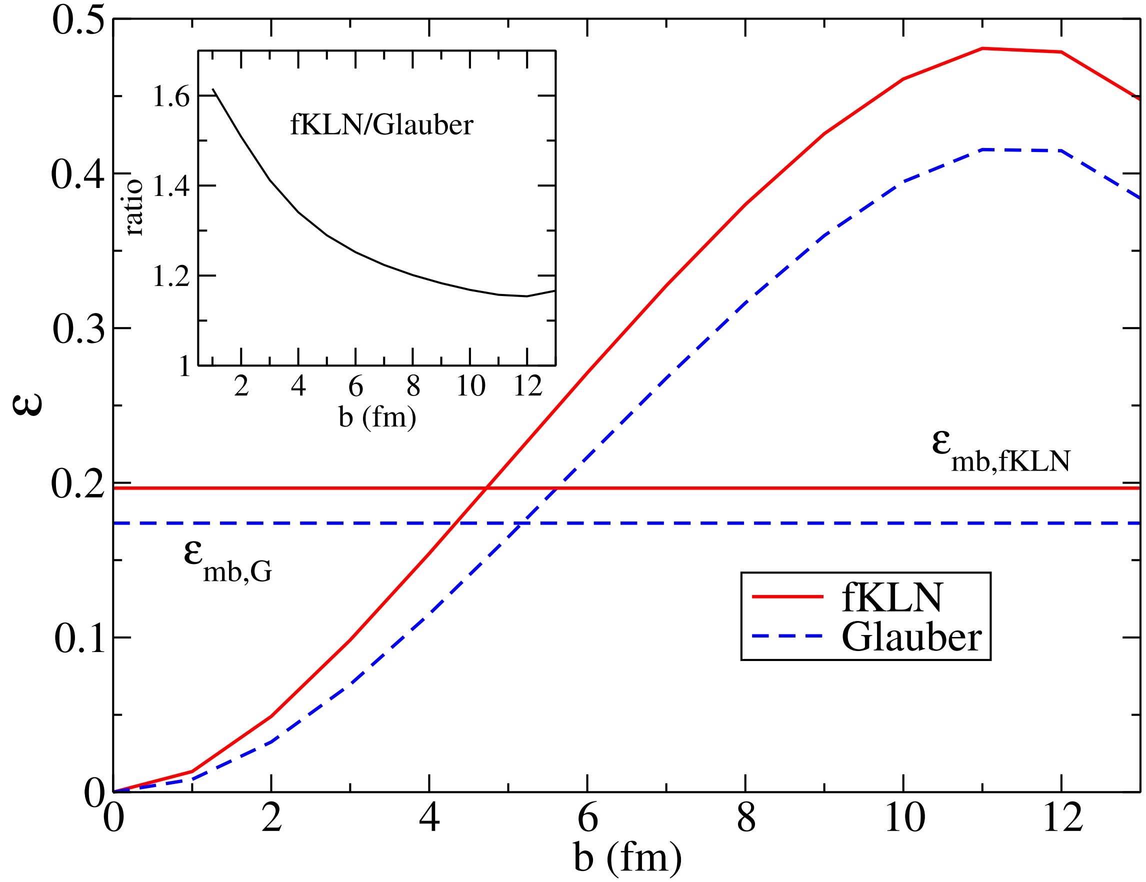
- **EoS:** EOS Q, EOS L, EOS L + chemical non-equilibrium HRG EOS

- **viscosities & relaxation times:** $\eta, \tau_\pi, \quad \zeta, \tau_\Pi$
- **initialization for** π^{mn}, Π

Uncertainties in the initial source eccentricity

(UH, Moreland, Song, arXiv:0908.2617)

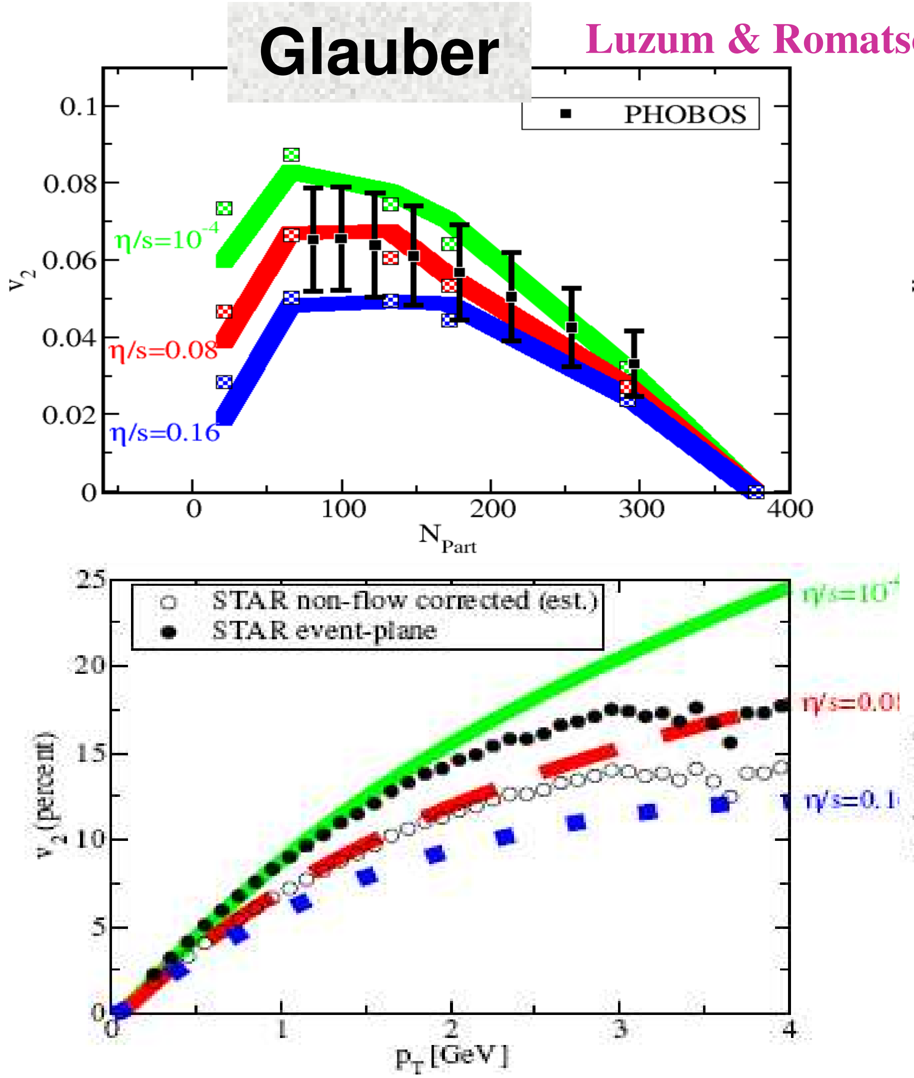
Initial source eccentricity vs. b

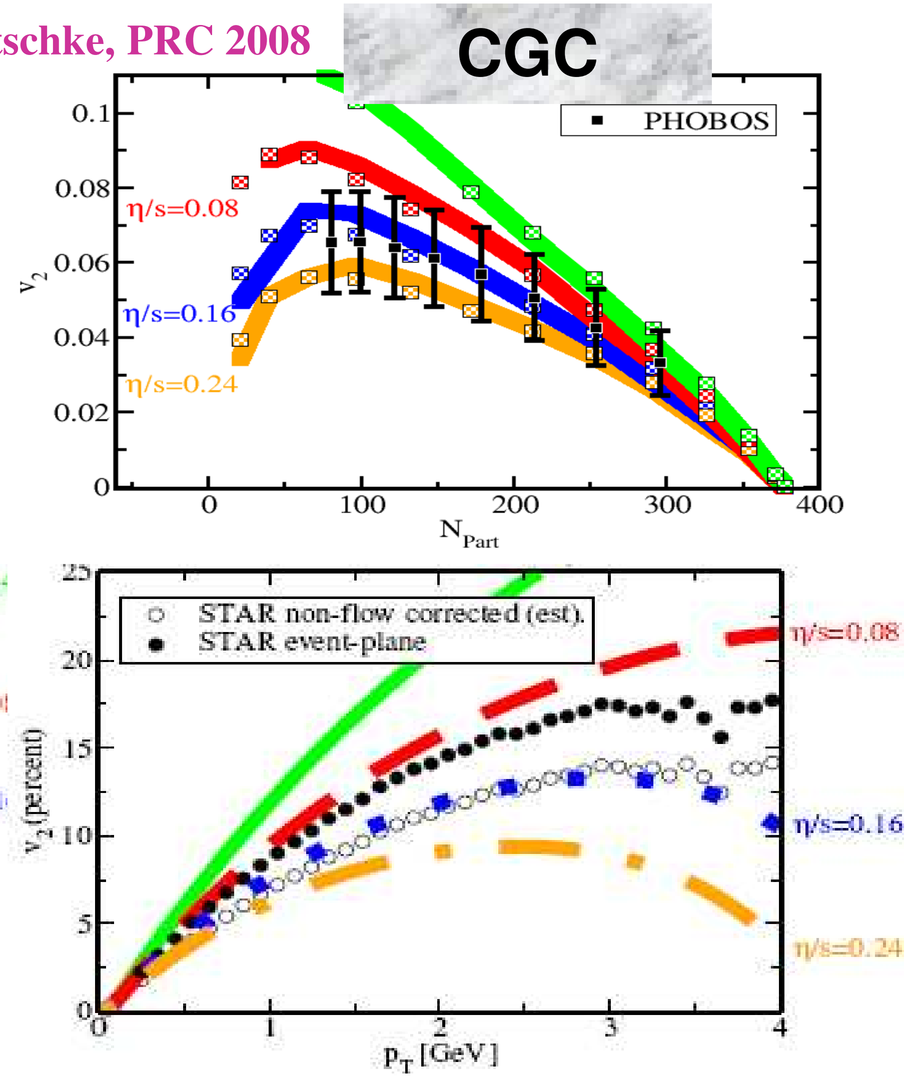


- CGC/fKLN model (Drescher et al.) gives larger source eccentricity than Glauber model, but excess is strongly centrality dependent!
- Minimum bias eccentricity 12% larger for fKLN than Glauber

Effect of **initial eccentricity** on v_2

Luzum & Romatschke, PRC 2008

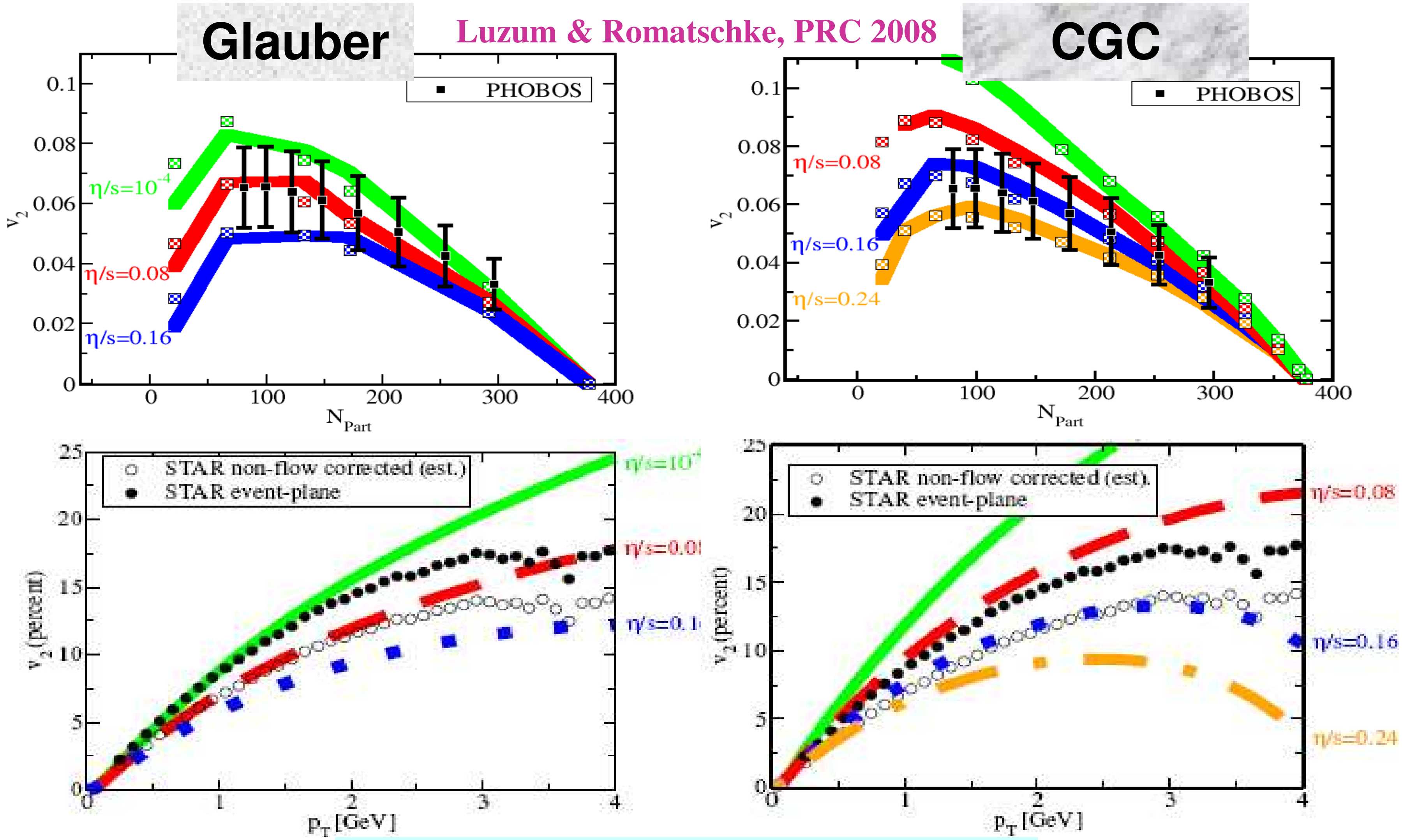




- Glauber vs.CGC: ~20-30% effect on $\mathbf{v_2}$ ⟺ ~100% uncertainty on η/s

Effect of **initial eccentricity** on v_2

Luzum & Romatschke, PRC 2008



$\eta / s < 0.16$?   NOT so fast !

- Effects from highly viscous & non-chemical equilibrium hadronic stage, bulk viscosity …

Extracting η/s from v_2 data

Input / parameters for viscous hydro

- initial conditions:
    - initial eccentricity

- treatment of hadronic stage and freeze-out procedure:
    - chemical composition of HRG
    - effects of highly viscous HRG

- EoS:

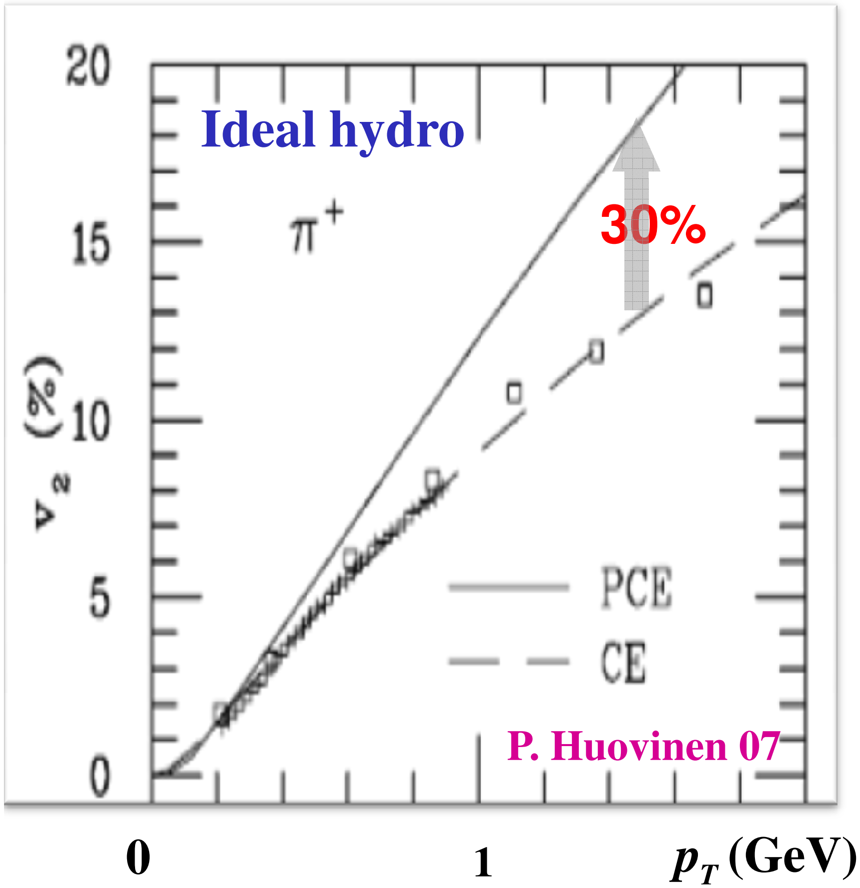
- viscosities & relaxation times:

- initialization for shear and bulk pressure

Effect of **HRG chemical composition** on v_2

PCE vs.CE (HRG)

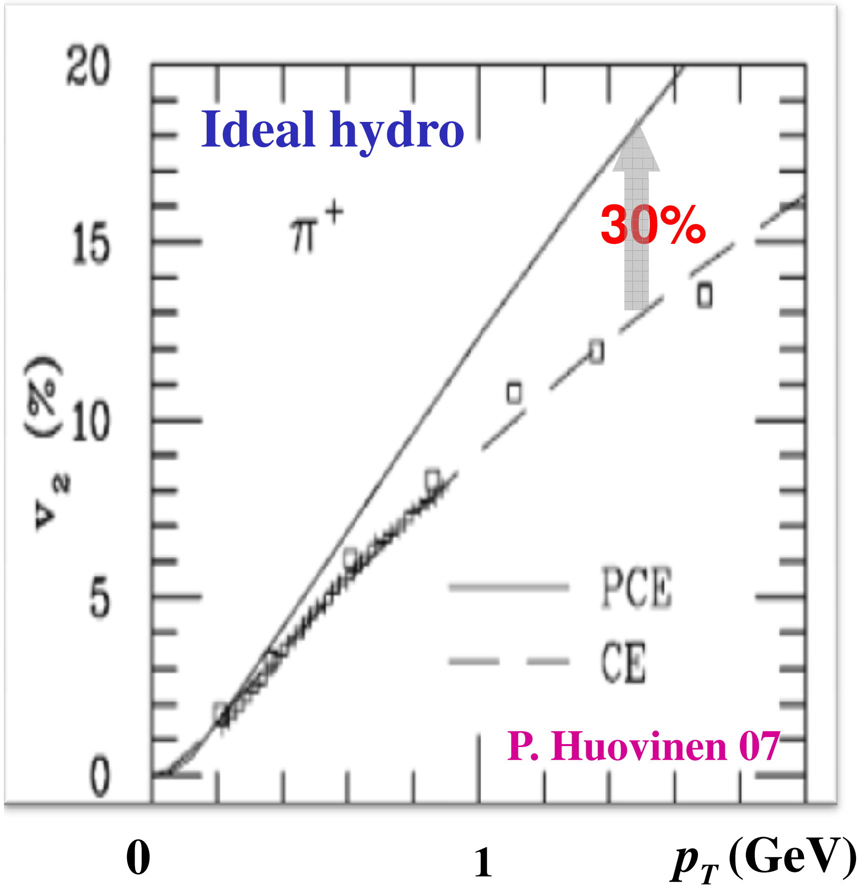
Partial Chemical Equilibrium (PCE)
vs. Chemical Equilibrium (CE)

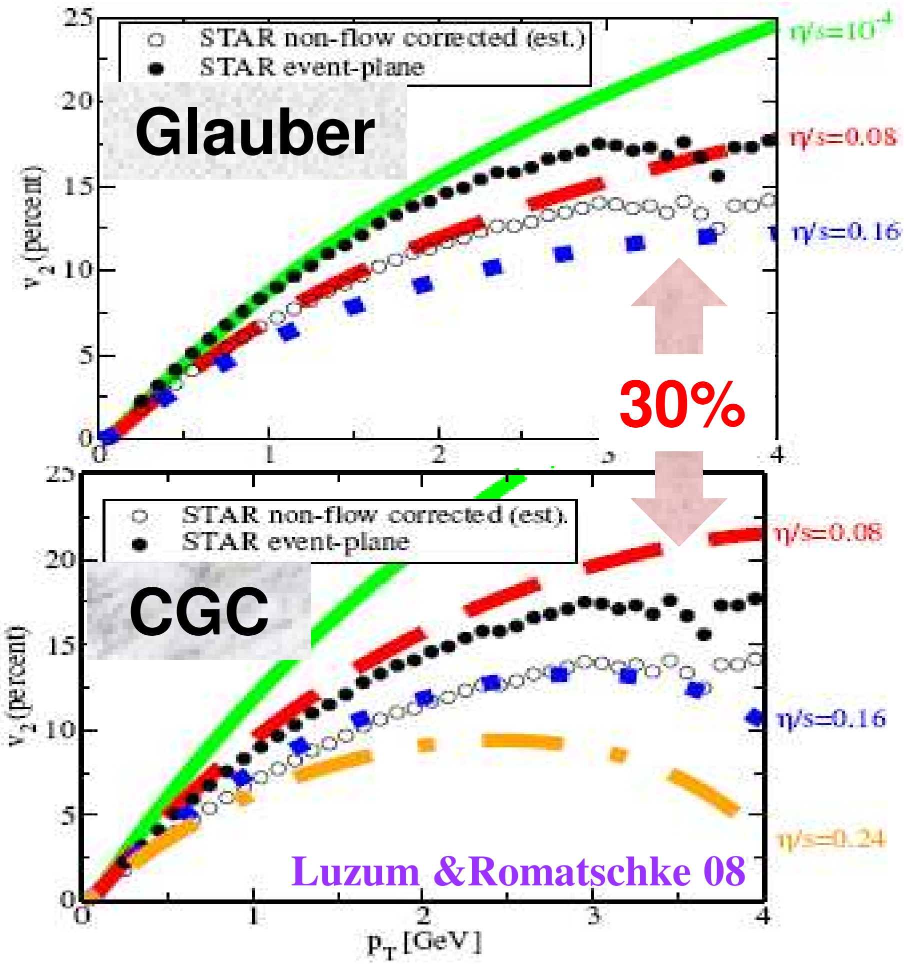


- PCE EoS vs. CE EoS (ideal hydro): changes v_2 by ~30%

Effects of **HRG chemical composition** on v_2

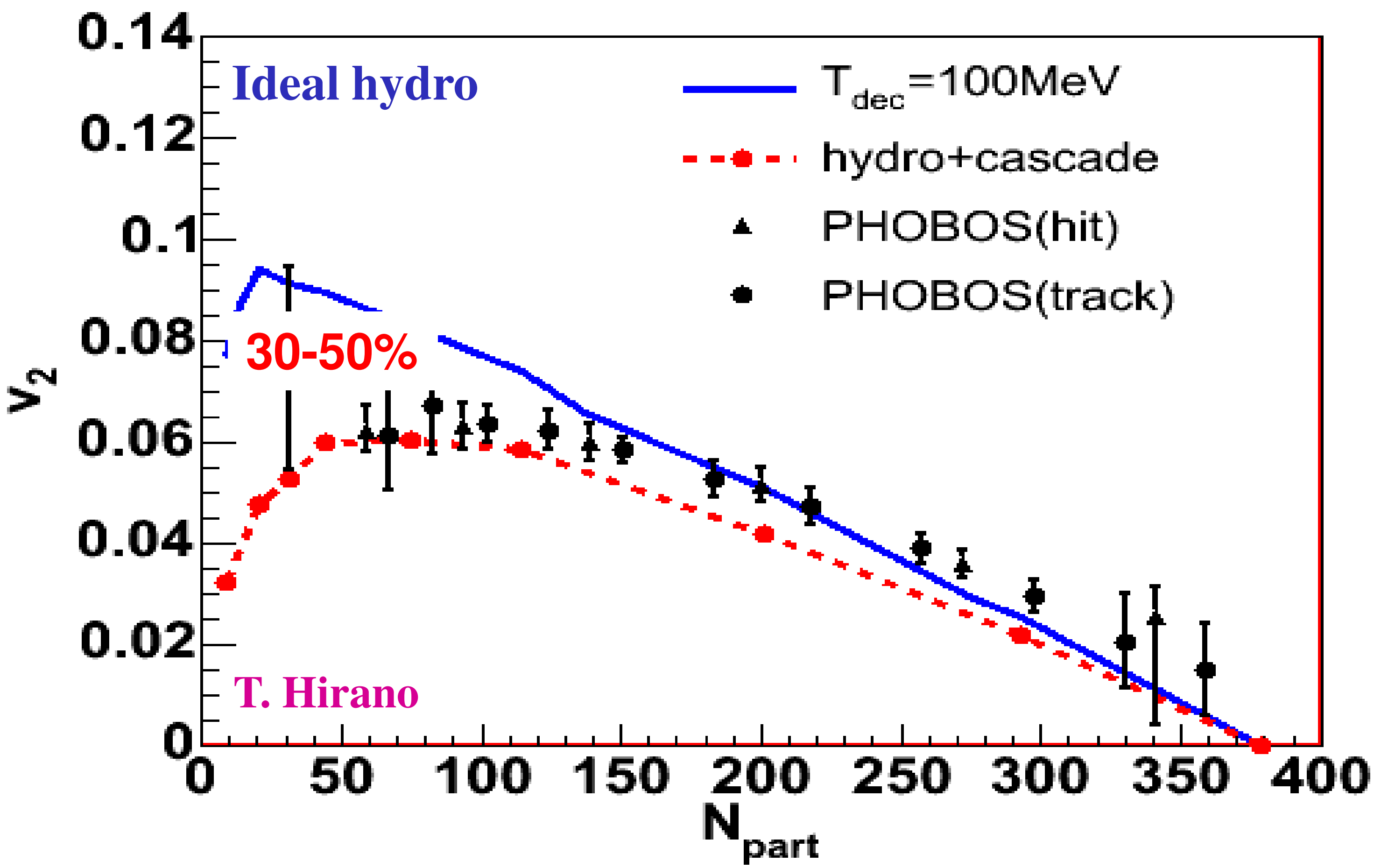
PCE vs.CE (HRG)





- PCE EoS vs. CE EoS (ideal hydro): changes v_2 ~30% ⟺ influences η/s ~100%
- Constraining η/s requires: **a proper description of partial chemical equilibrium in HRG**

Effects of highly **viscous hadronic stage** on v_2



- highly viscous hadronic stage: changes v_2 ~**30-50%** ⟺ influences η/s ~**100-150%**
- need viscous hydro + hadron cascade hybrid approach

Extracting η/s from v_2 data

Input / parameters for viscous hydro

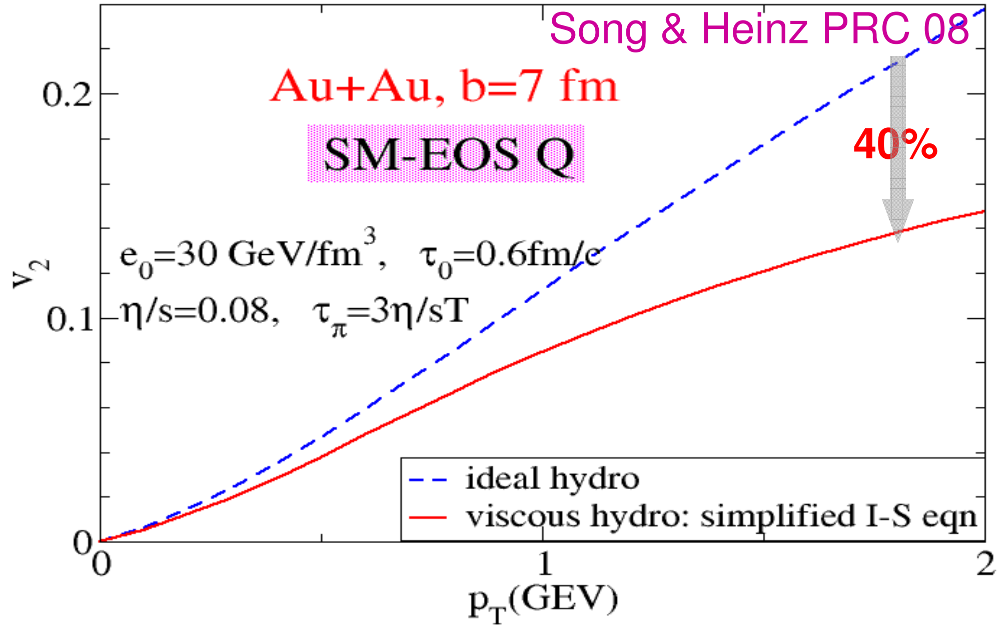
- initial conditions:
    - initial eccentricity
- treatment of hadronic stage and freeze-out procedures:
    - chemical composition of HRG
    - viscosity of HRG
- **EoS:** (EOS Q vs. EOS L)
- viscosities & relaxation times:
- initialization for shear and bulk pressure

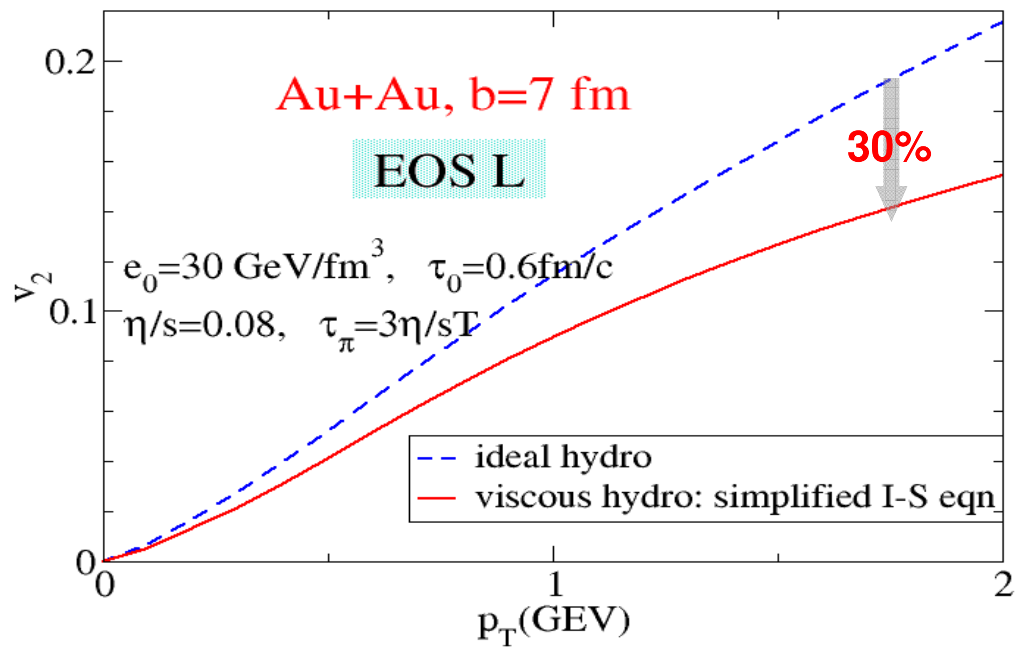
Effects from softness of EOS

SM-EOS Q vs. EOS L

– softness of EOS: ~5-10% effects on v_2 ⟺ ~25% uncertainties on η/s

– EOS L: Katz 05 lattice data for QGP + CE EOS for HRG





in progress (with Shen Chun & Pasi Huovinen):

– more realistic EOS for hydro

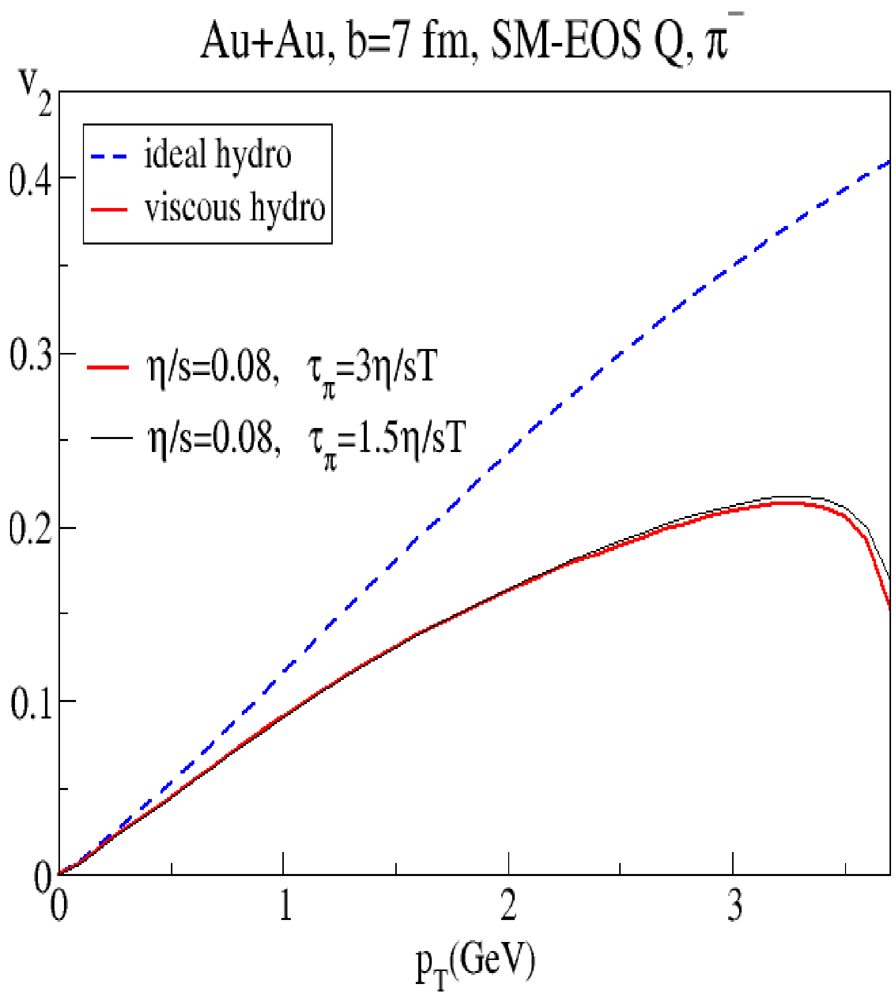
0903 lattice data for QGP + **PCE EOS** for HRG

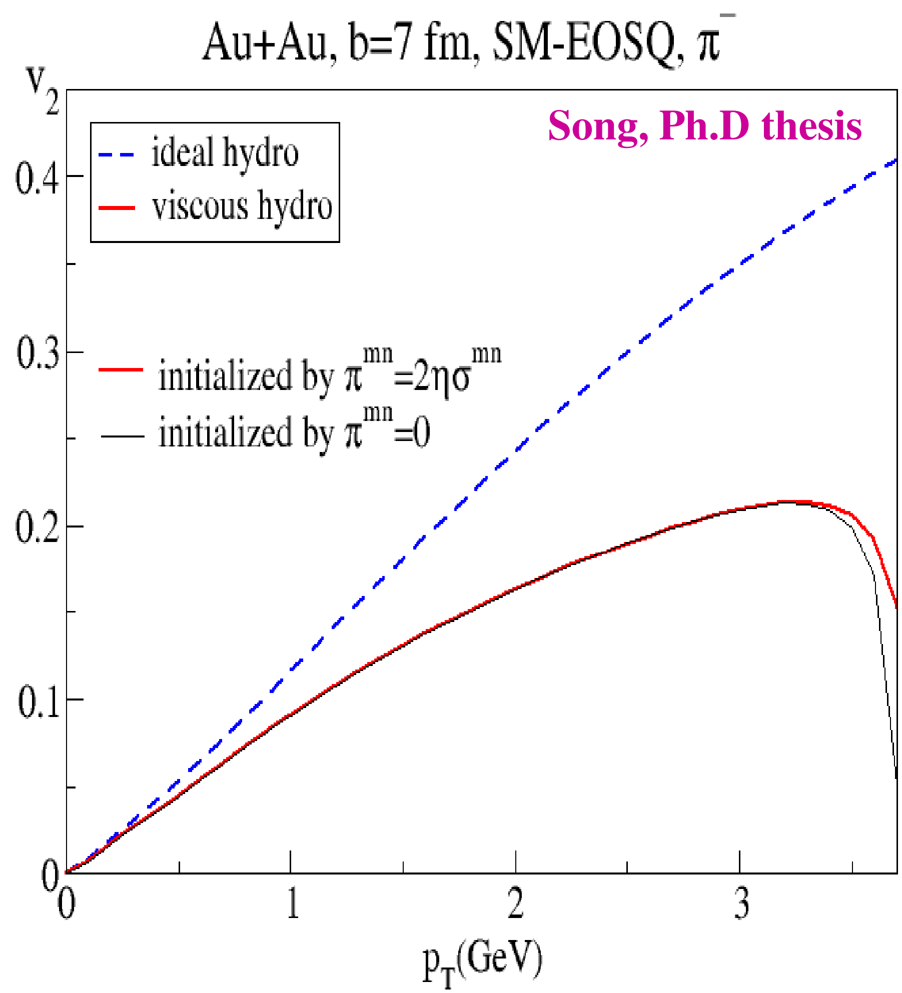
Extracting η/s from v_2 data

Input / parameters for viscous hydro

- initial conditions:
  - initial eccentricity
- treatment of hadronic stage and freeze-out procedures:
  - chemical composition of HRG
  - viscosity of HRG
- EoS: (EOSQ vs EOSL)
- **viscosities & relaxation times:** $\eta, \tau_\pi, \quad \zeta, \tau_\Pi$
- **initialization for shear and bulk pressure**

shear pressure: relaxation times & initialization





- v_2 is insensitive to initializations of π^{mn} and relaxation time τ_π ! (since τ_π is short)
- when extracting η/s: one can neglect the uncertainties from τ_π & initialization of π^{mn}

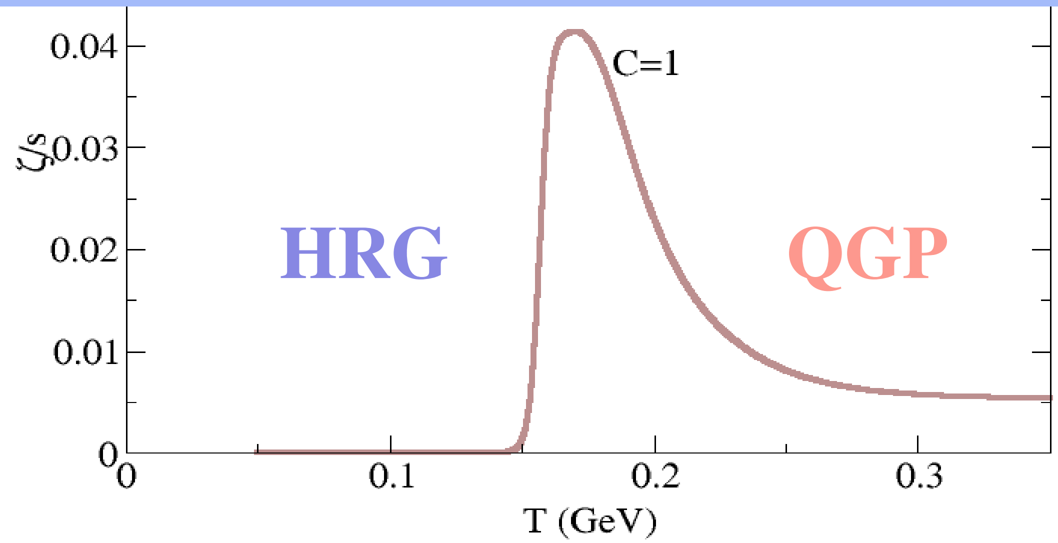
Extracting η/s from v_2 data

Input / parameters for viscous hydro

- **initial conditions:**
  - **initial eccentricity**
- **treatment of hadronic stage and freeze-out procedures:**
  - **chemical composition of HRG**
  - **viscosity of HRG**
- **EoS:** (EOSQ vs EOSL)
- **shear pressure: relaxation times and initialization**
- **effects from bulk viscosity**

bulk viscosity and relaxation time

Bulk viscosity:

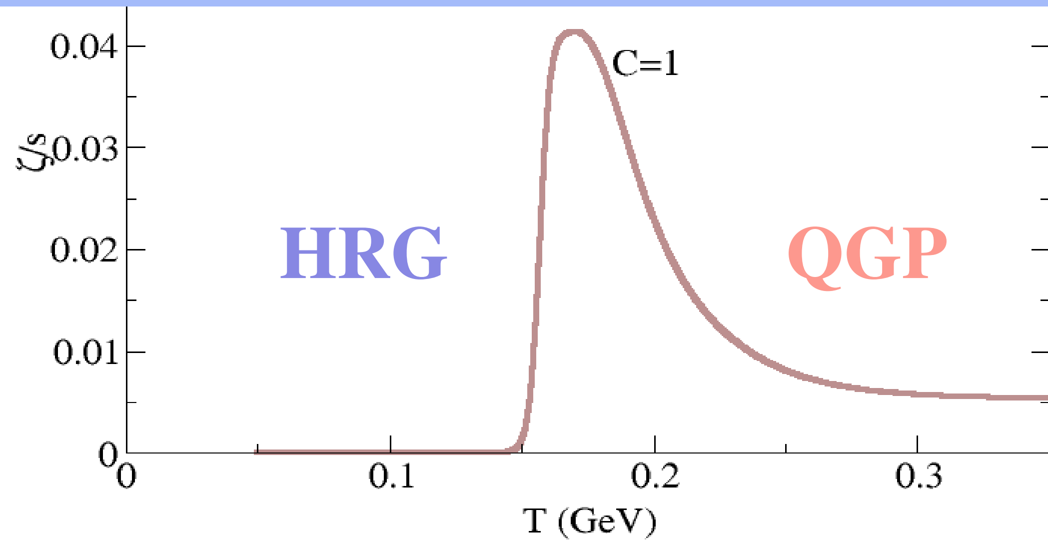


Relaxation times: $\tau_\Pi \sim \zeta$ also peaks near T_c,

this plays an important role for bulk viscous dynamics

bulk viscosity and relaxation time

Bulk viscosity:



Relaxation times: $\tau_\Pi \sim \zeta$ also peaks near T_c,

this plays an important role for bulk viscous dynamics

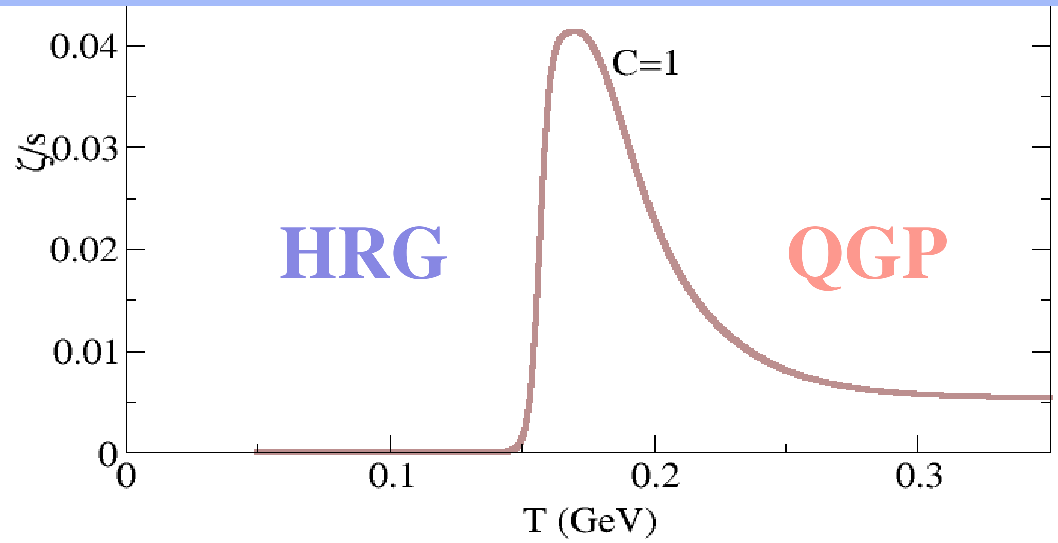
Zero initialization: $\Pi_0 = 0$

large τ_Π near T_c $\longrightarrow$ **prevents Π from growing to large values**

$\longrightarrow$ **suppresses bulk viscous effects**

bulk viscosity and relaxation time

Bulk viscosity:



Relaxation times: $\tau_\Pi \sim \zeta$ also peaks near T_c,

this plays an important role for bulk viscous dynamics

N-S initialization: $\Pi_0 = -\zeta(\partial \cdot u)$

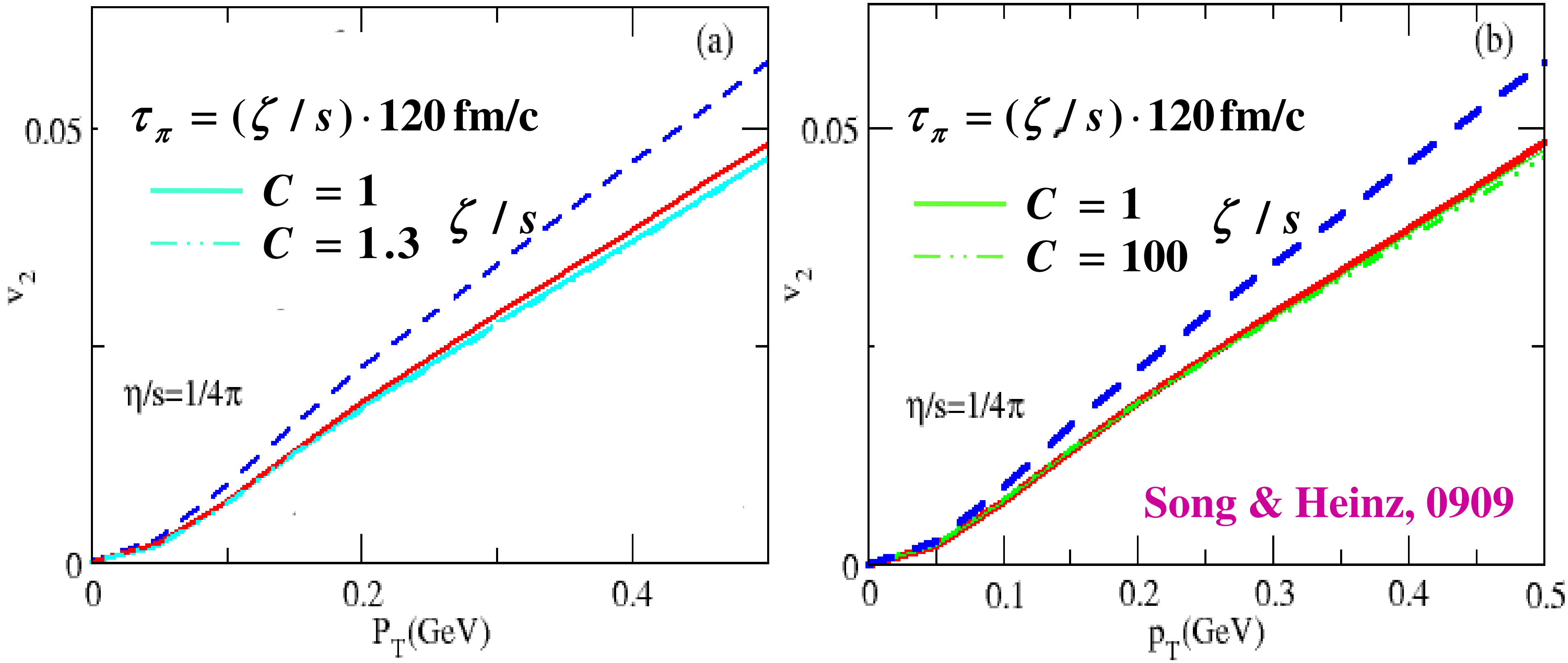
large τ_Π near T_c $\longrightarrow$ **keeps large negative value of** Π in phase transition region

$\longrightarrow$ **viscous hydro breaks down** $(p + \Pi < 0)$ for larger ζ/s

viscous hydro is only valid with small ζ/s $\longrightarrow$ **small bulk viscous effects on** v_2

Uncertainties from **bulk viscosity**

N-S initialization | Zero initialization



- with critical slowing down of τ_Π, effects from bulk viscosity are much smaller than from shear viscosity

bulk viscosity influences v_2 **~5%** (N-S initial.) **<4%** (zero initial.)
⟺ uncertainties to η/s **~20%** (N-S initial.) **<15%** (zero initial.)

Extracting η / s from RHIC data

--the current status of viscous hydrodynamics

(uncertainties in v_2)

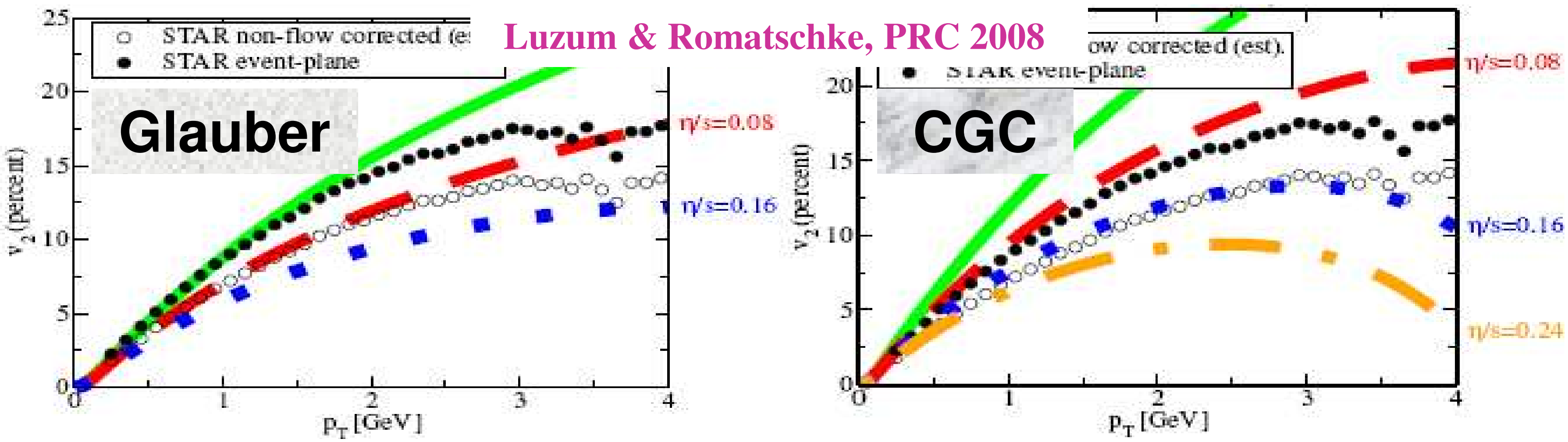
-initial conditions: CGC vs. Glauber **~20-30%**

-EoS: EOS Q, vs. EOS L **~5-10%**

-chemical composition of HRG : (PEC vs. CE) **~30%**

-viscosity of HRG (or equil. HRG vs. non-equil. HRG) **: ~30-50%**

-bulk viscosity: ~5%



(uncertainties in η / s)

-initial conditions: CGC vs. Glauber **~100%**

-EoS: EOS Q, vs. EOS L **~25%**

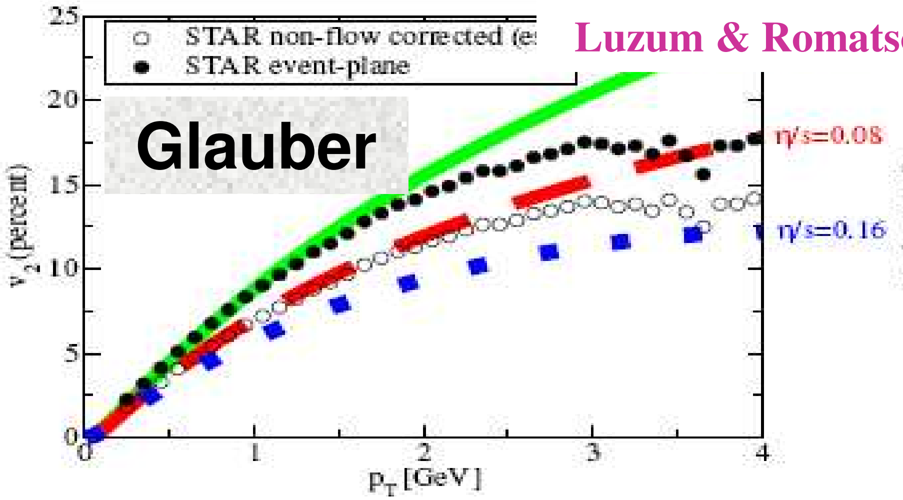
-chemical composition of HRG : (PEC vs. CE) **~100%**

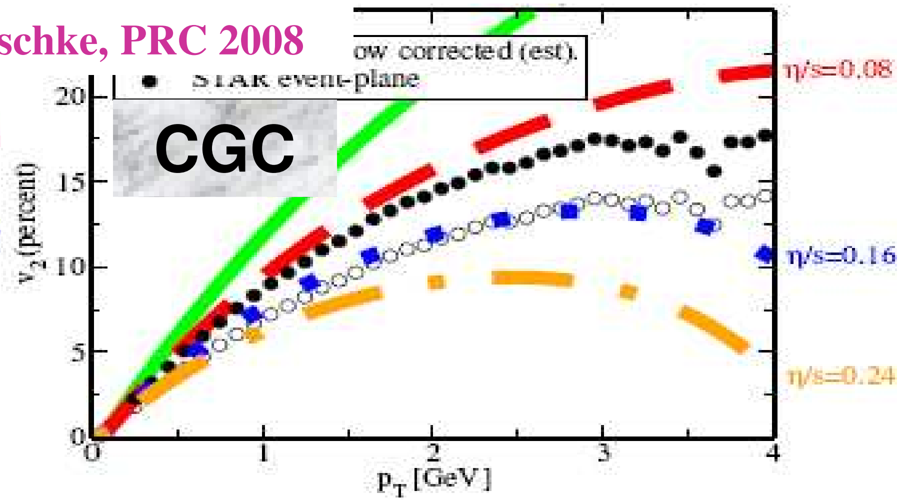
-viscosity of HRG (or equil. HRG vs. non-equil. HRG) **: ~100-150%**

-bulk viscosity: ~20%

conservative upper limit: $\eta / s \leq 5 \times (1/4\pi)$

Luzum & Romatschke, PRC 2008



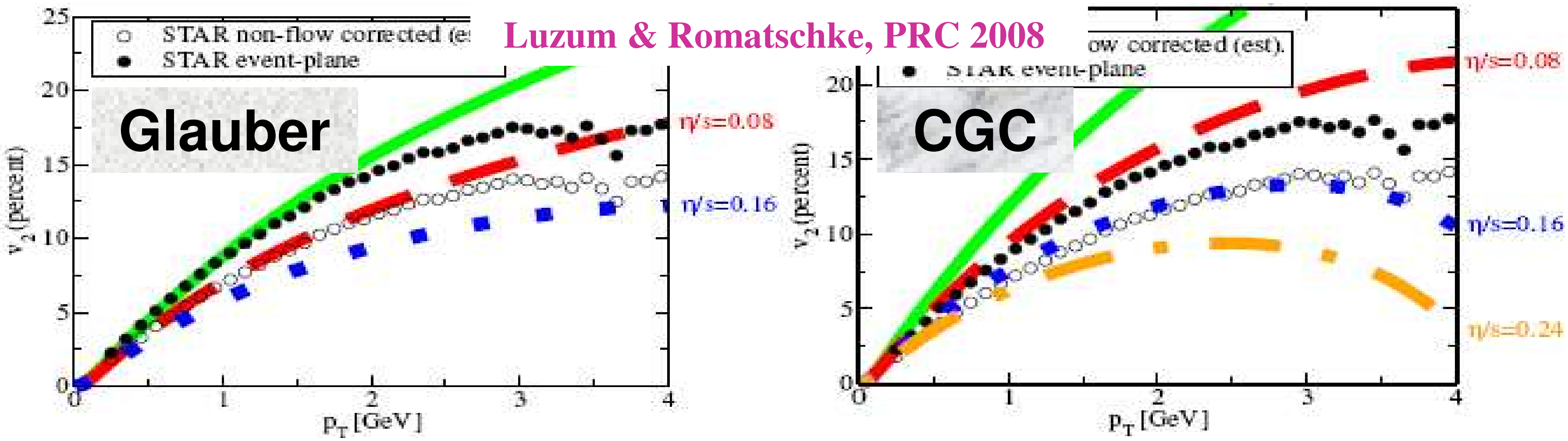


(uncertainties in η/s)

- **-initial conditions:** CGC vs. Glauber **~100%** **recent progress: Heinz et al. 0907**
- **-EoS:** EOS Q, vs. EOS L **~25%** ✓
- **-chemical composition of HRG :** (PEC vs. CE) **~100%** ✓
- **-viscosity of HRG** (or equil. HRG vs. non-equil. HRG) **: ~100-150%** ✓
- **-bulk viscosity: ~20%**

conservative upper limit: $\eta/s \leq 5\times(1/4\pi)$

Can we further increase the accuracy of extracted η/s ? *YES !*



Luzum & Romatschke, PRC 2008

(uncertainties in η/s)

-initial conditions: CGC vs. Glauber ~100% recent progress: Heinz et al. 0907

-EoS: EOS Q, vs. EOS L ~25% ✓

-chemical composition of HRG : (PEC vs. CE) ~100% ✓

-viscosity of HRG (or equil. HRG vs. non-equil. HRG) : ~100-150% ✓

-bulk viscosity: ~20%

conservative upper limit: $\eta / s \leq 5 \times (1/4\,\pi)$

-to further decrease the uncertainties from bulk viscosity (or to separate shear and bulk viscosity from exp. data), **one needs more sensitive exp. observables**

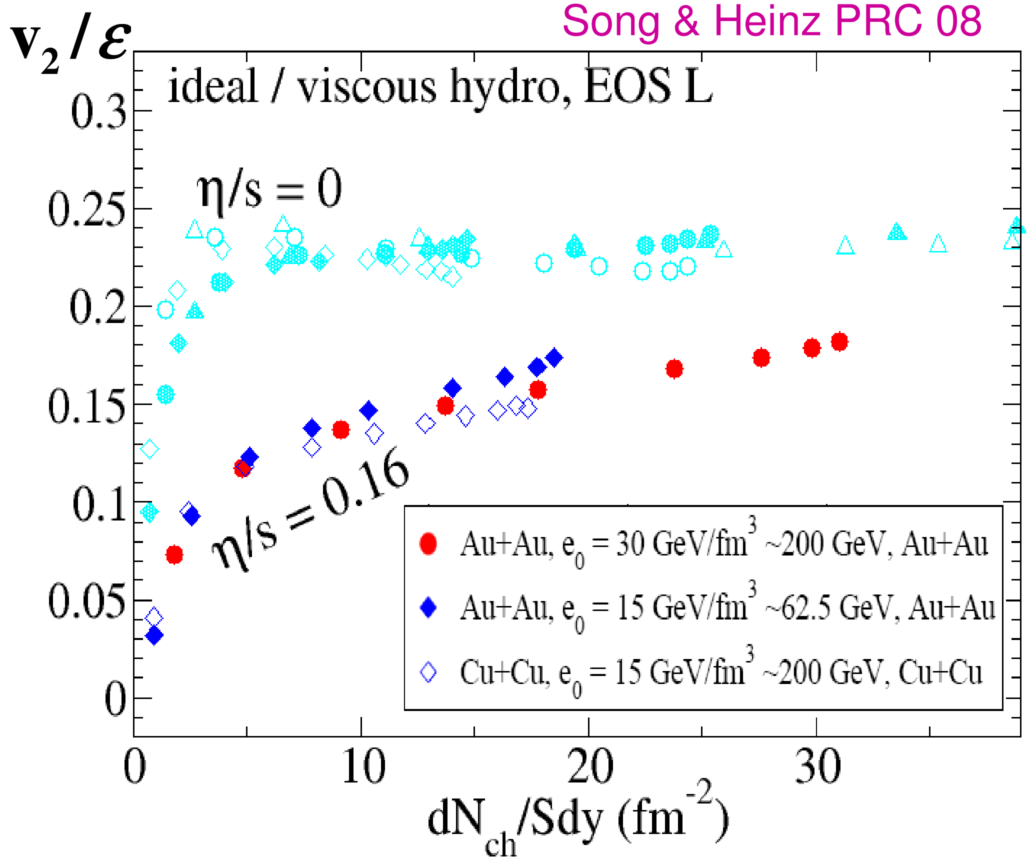
Other observables that are sensitive to η/s:

- v_2/ε: dependence on system size & shear viscosity
- photon spectra
- HBT radii

... ...

System size dependence of shear viscous effects

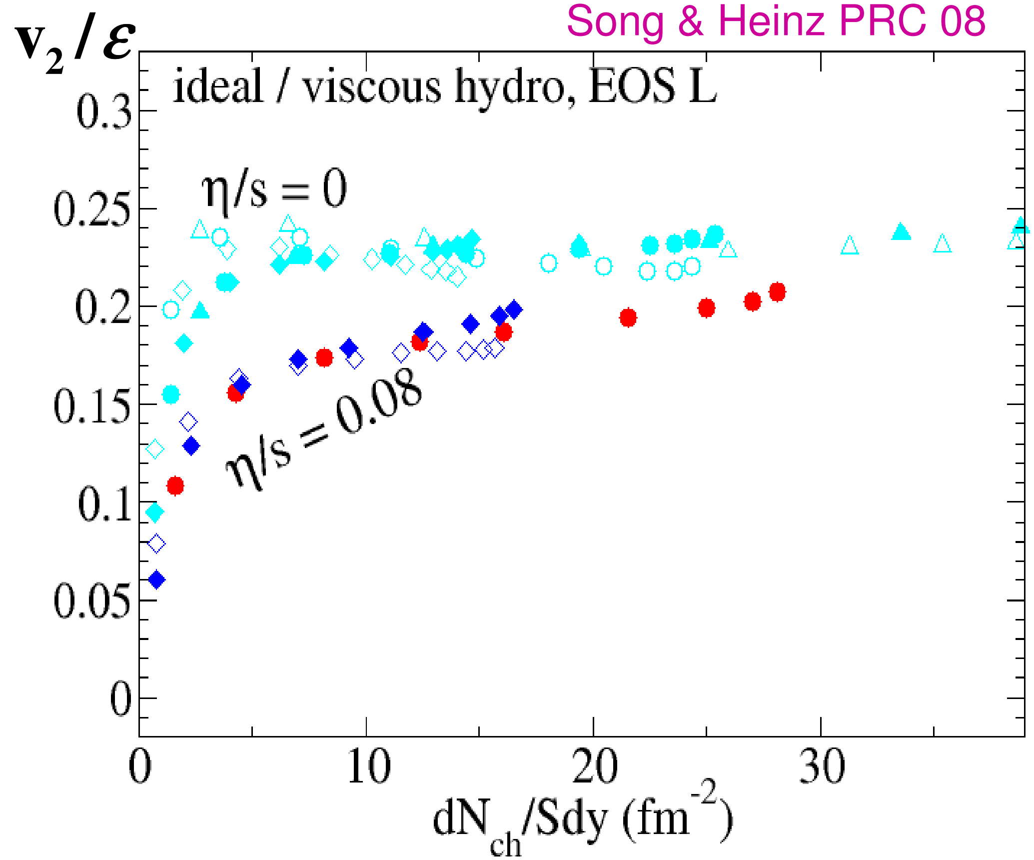
Song & Heinz PRC 08



- larger shear viscous suppression of elliptic flow in smaller systems

System size dependence of shear viscous effects

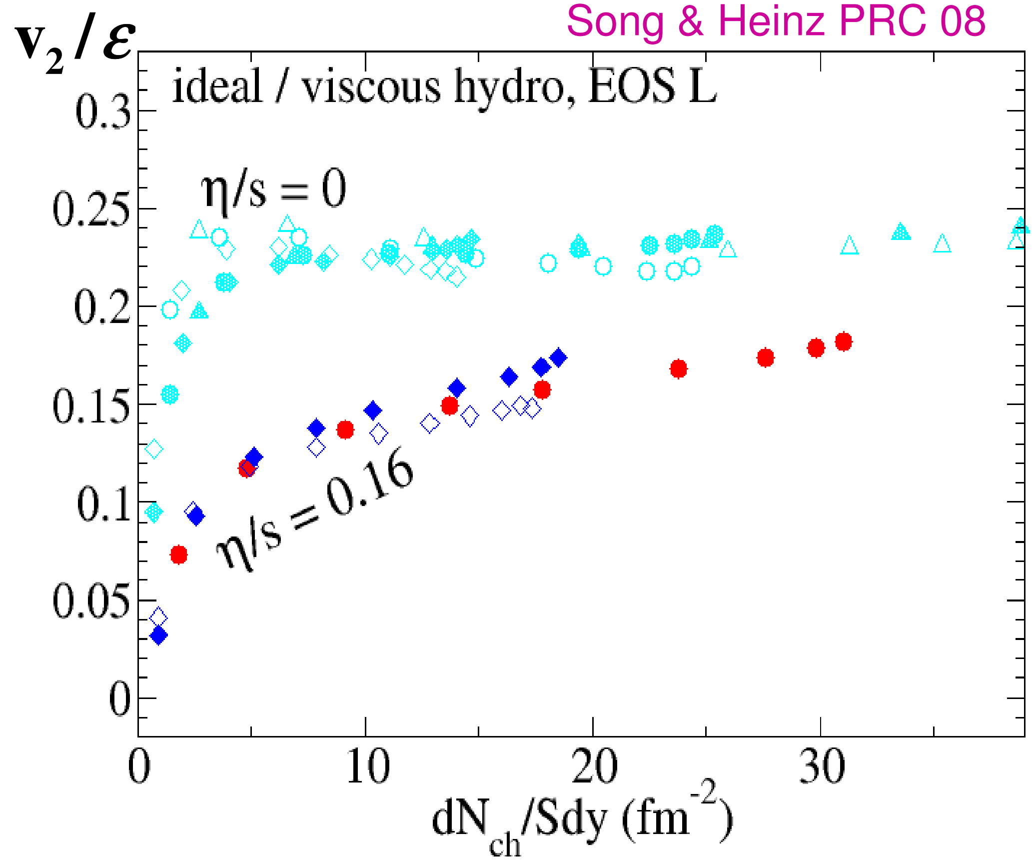
Song & Heinz PRC 08



- larger shear viscous suppression of elliptic flow in smaller systems
- the slope of the multiplicity scaling of v_2/ε is sensitive to the value of shear viscosity

System size dependence of shear viscous effects

Song & Heinz PRC 08



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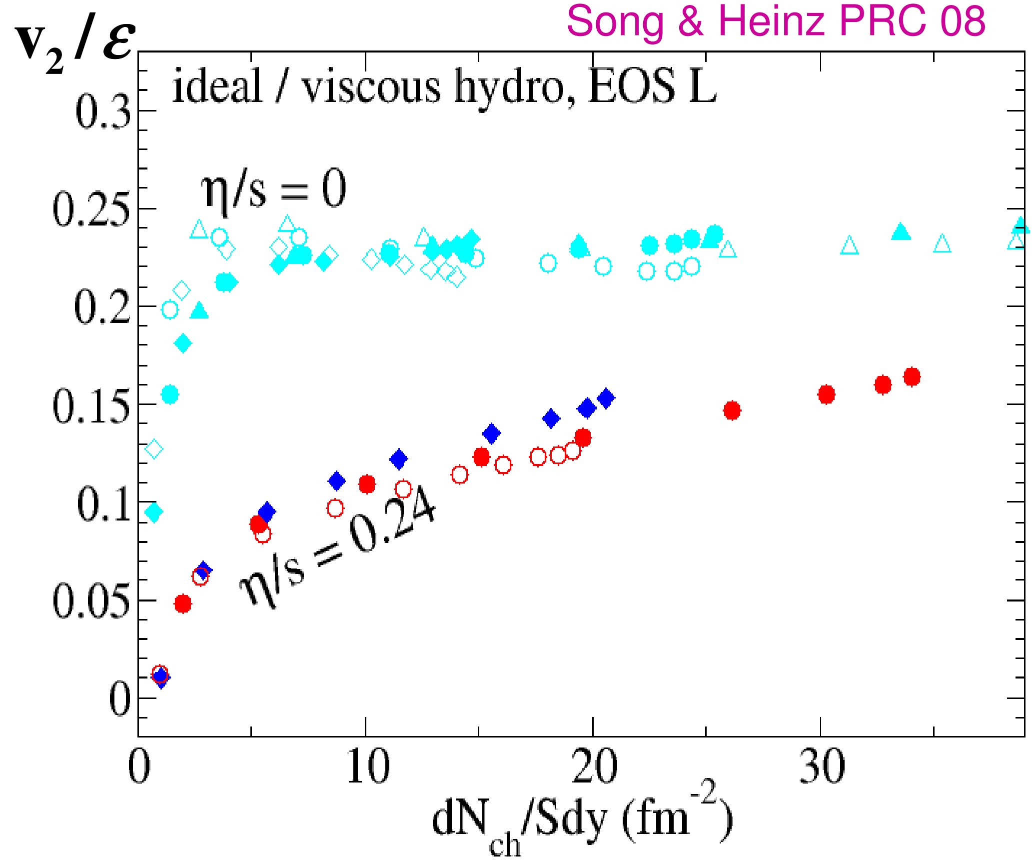
System size dependence of shear viscous effects

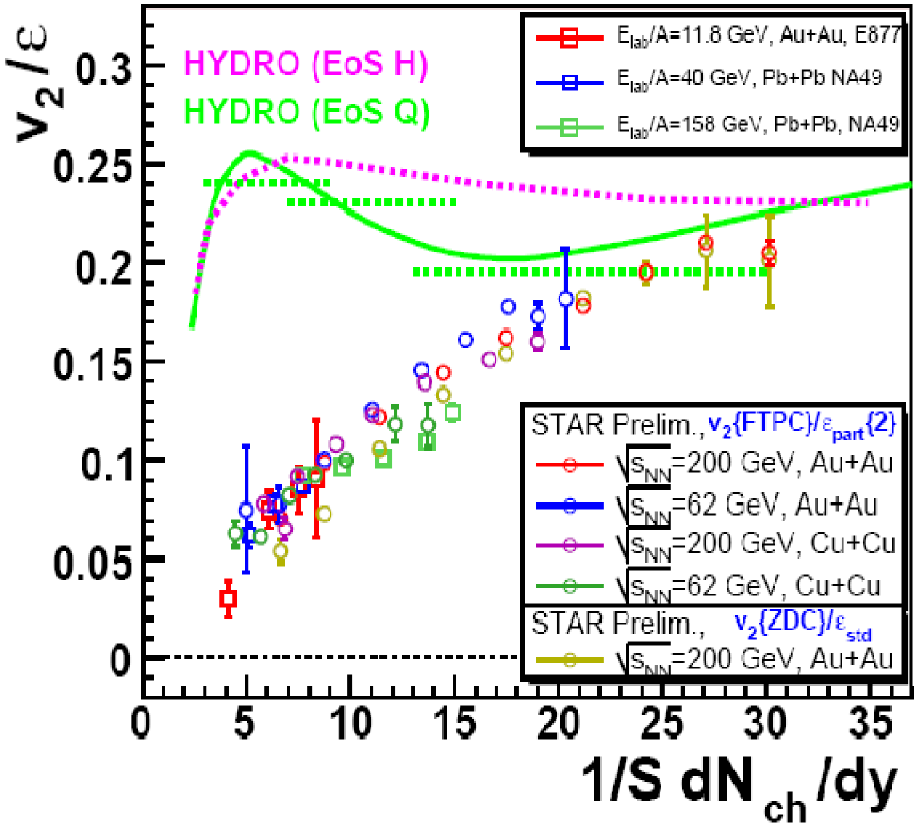
Song & Heinz PRC 08

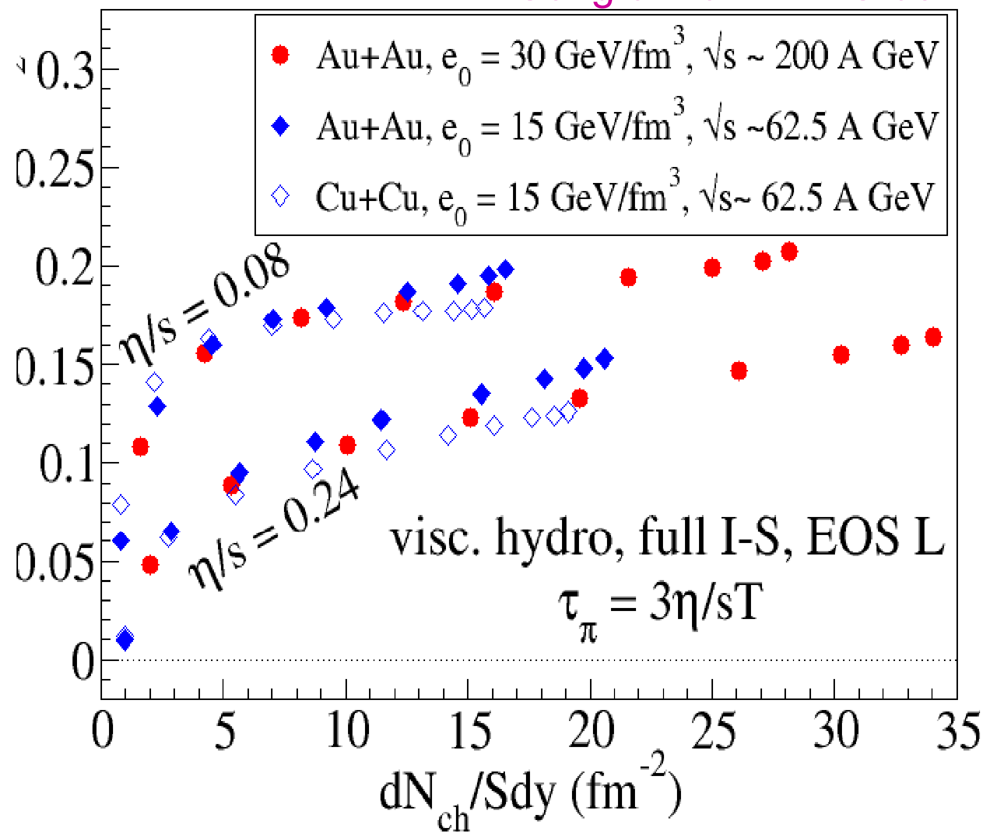


- larger shear viscous suppression of elliptic flow in smaller systems
- the slope of the multiplicity scaling of v_2/ε is sensitive to the value of shear viscosity

Multiplicity scaling of v_2/ε

Song & Heinz PRC 08

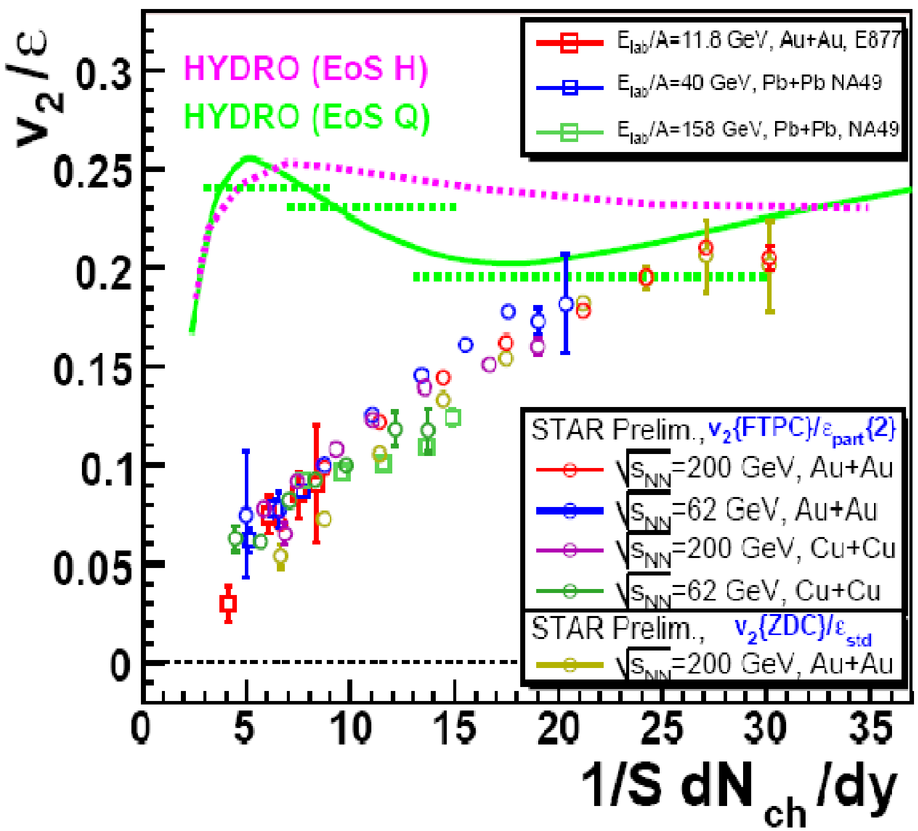


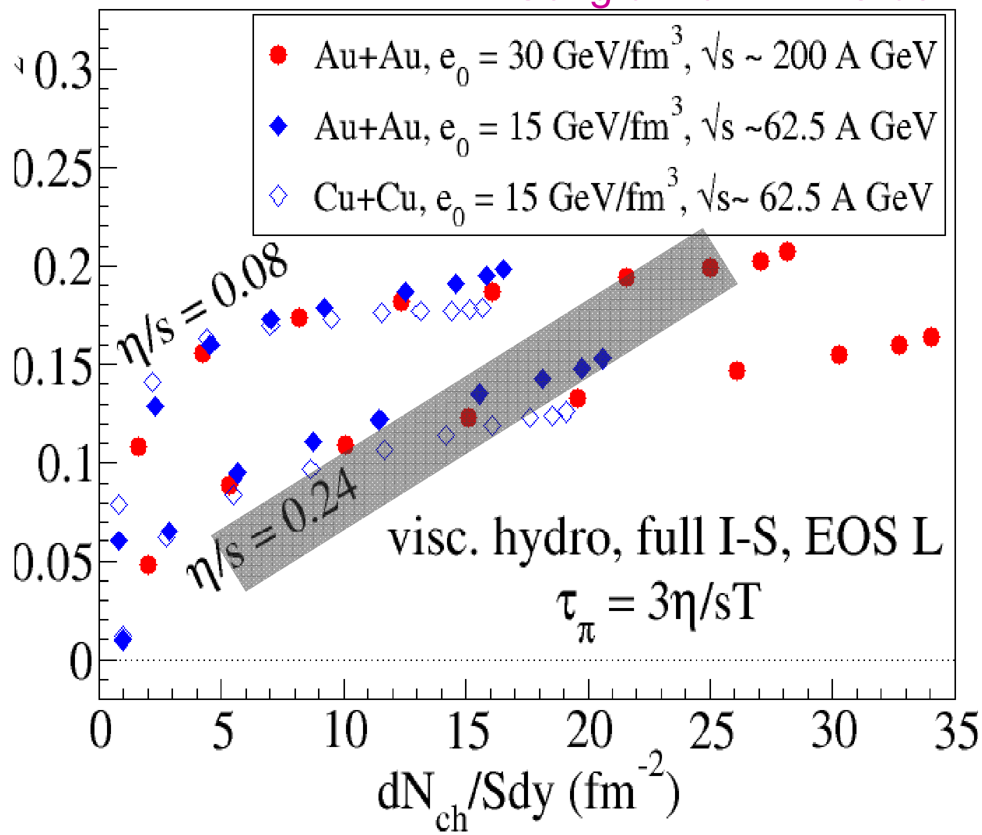


- the slope of the multiplicity scaling of v_2/ε is sensitive to the value of shear viscosity
- to reproduce the experimental data (slope), a constant η/s is not enough!

Multiplicity scaling of v_2/ε

Song & Heinz PRC 08





- the slope of the multiplicity scaling of v_2/ε is sensitive to the value of shear viscosity
- to reproduce the experimental data (slope), a constant η/s is not enough!
- data indicate: smaller viscous effects in QGP; larger viscous effects in HRG
- need **viscous hydro + hadron cascade** hybrid approach or inputting $\eta/s(T)$

A short summary

- v_2 is sensitive to η/s

A first attempt to constrain η/s from RHIC data indicates

$$\eta/s \le 5\times(1/4\pi)$$

BUT:
to extract QGP viscosity, one must consider (at least) all the following aspects:

- a realistic EOS: EOS L vs. SM-EOS Q, PCE vs. CE
- initial conditions: CGC vs. Glauber initialization, optical vs. fluctuations
- bulk viscosity: uncertainties from bulk viscosity
- hadronic stage : viscous hydro+ hadron cascade

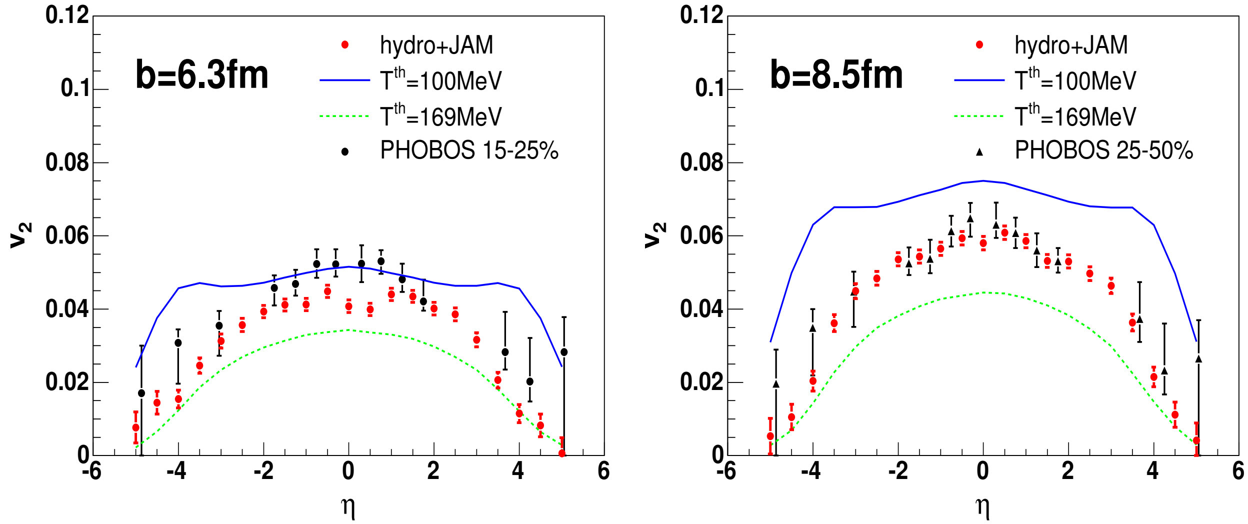
…… ……

Supplements

Late hadronic dissipation explains reduced v_2 at forward rapidity and in peripheral collisions:

3D Hydro+Cascade Model: Ideal fluid dynamics for QGP above T_c, hadronic cascade with realistic cross sections (JAM) below T_c

T. Hirano, U. Heinz, D. Kharzeev, R. Lacey, Y. Nara, PLB 636 (2006) 299



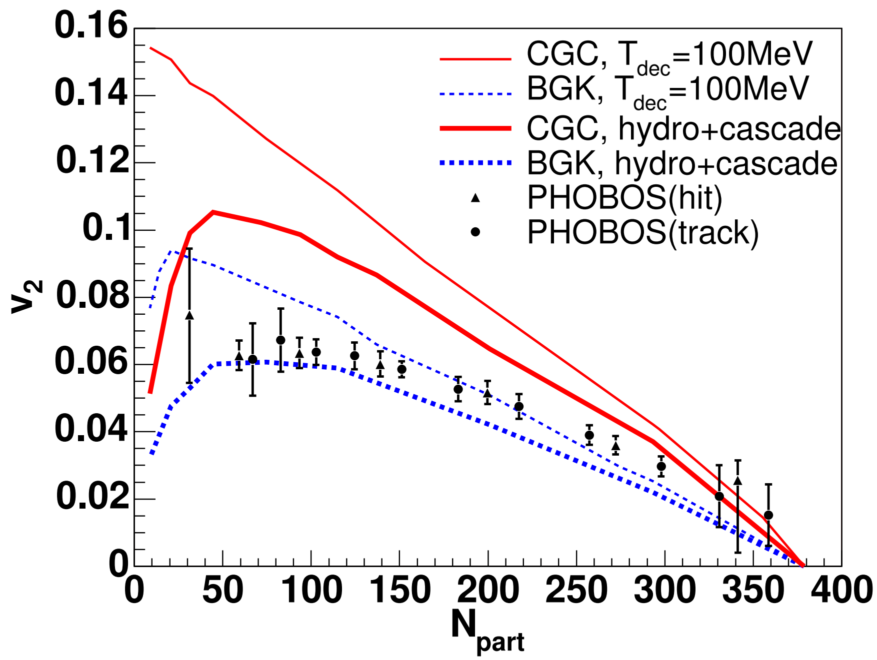
[Glauber model initial conditions (85% soft/15% hard)]

- Not enough elliptic flow from perfect QGP fluid – some hadronic contribution to v_2 is required
- Treating the hadronic stage as ideal fluid overpredicts v_2 in peripheral collisions and at forward rapidities
- Dissipation in hadronic cascade brings theory in line with data
- $\Longrightarrow$ No need for QGP viscosity!? Only if you trust Glauber!

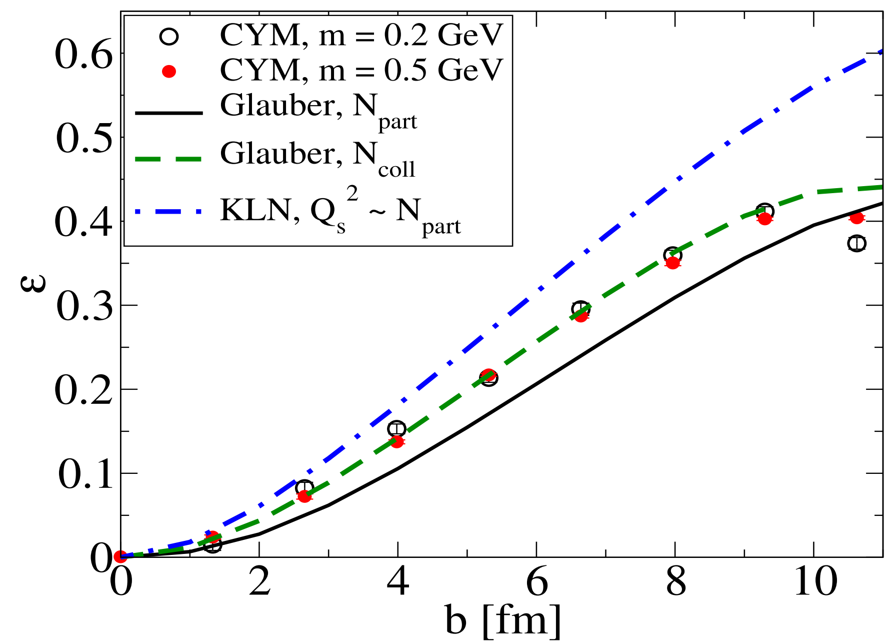
But: CGC gives larger initial eccentricity!

3D Hydro+Cascade Model: Ideal fluid dynamics for QGP above T_c, hadronic cascade with realistic cross sections (JAM) below T_c

Hirano et al., PLB 636 (2006) 299



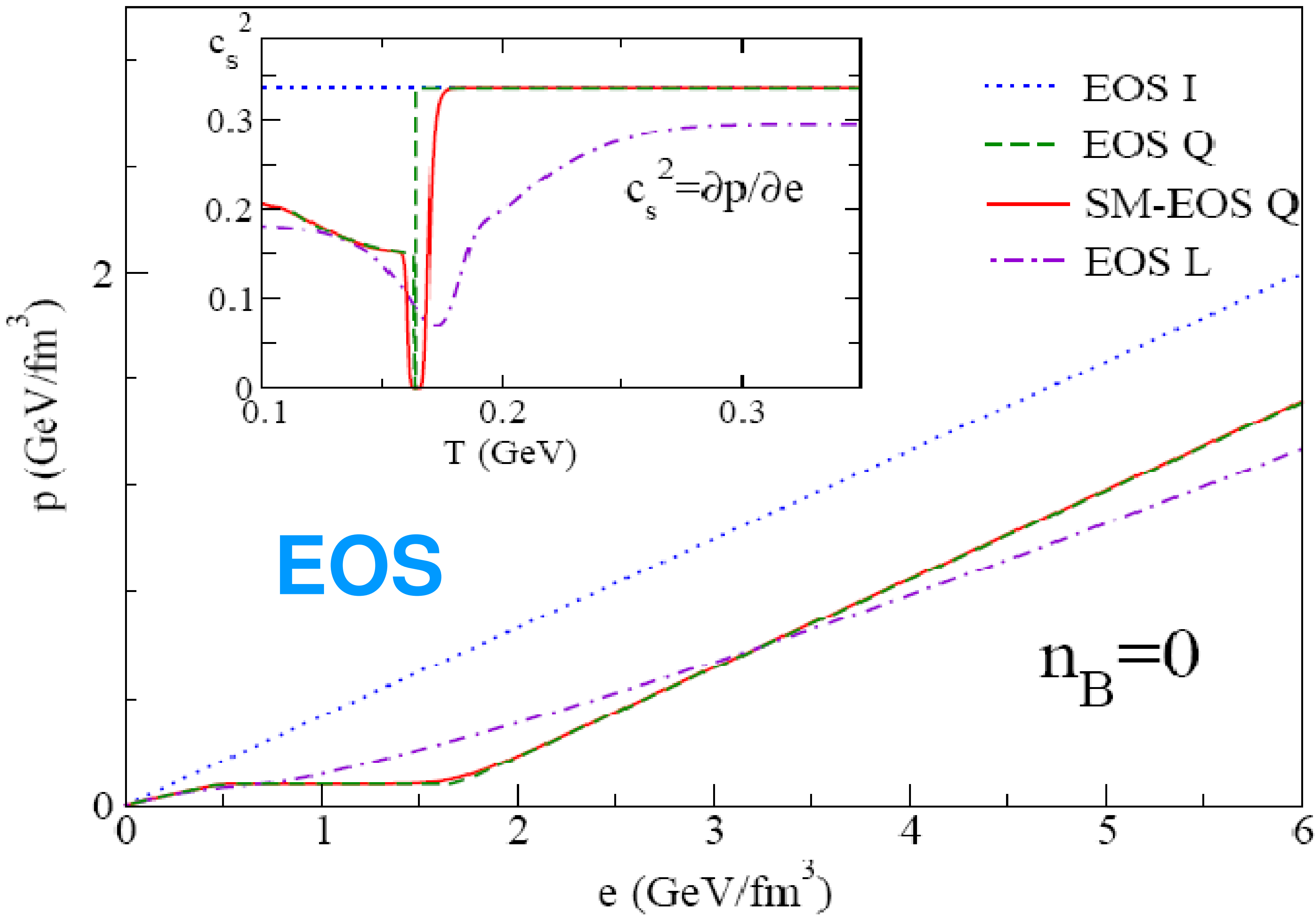
Lappi & Venugopalan, PRC 74 (2006) 054905



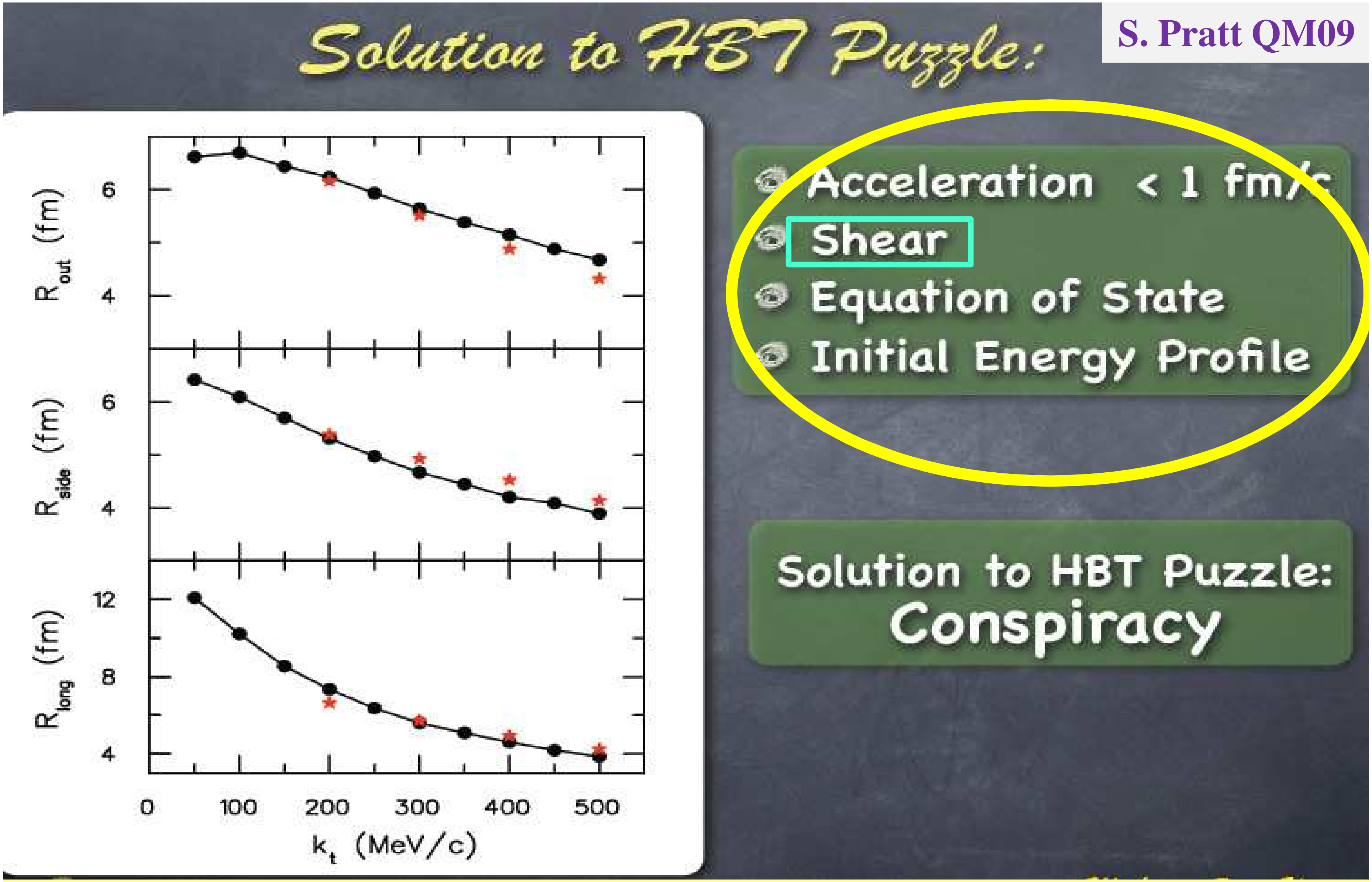
- Hadronic dissipation reduces elliptic flow buildup in peripheral collisions
- Color Glass Condensate (CGC-KLN) model (McLerran & Venugopalan 1994; Kharzeev, Levin, Nardi 2001) produces steeper edge of initial distribution, resulting in larger eccentricities ϵ than in Glauber model
- Ideal hydrodynamics turns larger spatial eccentricity ϵ into larger elliptic flow v_2
- For Glauber model initial conditions, hadronic dissipation fully explains the data; for CGC/KLN initial conditions hadronic dissipation not enough – need additional QGP viscosity!

$\Longrightarrow$ To isolate effects from early viscosity, need better control over initial conditions!

EOS



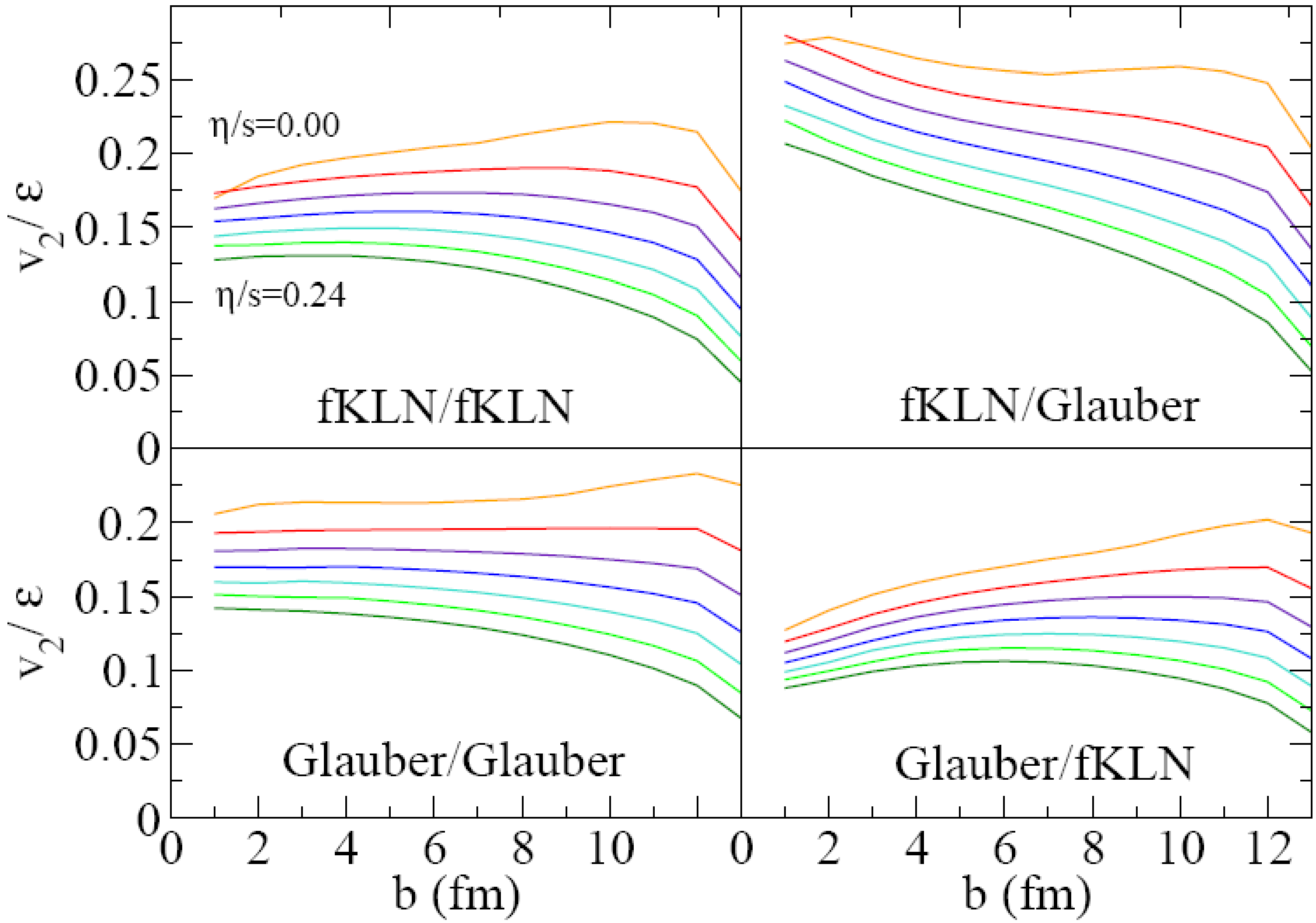
c_s^2
0.3
0.2
0.1
0
0.1
0.2
0.3
T (GeV)
$c_s^2 = \partial p / \partial e$
EOS I
EOS Q
SM-EOS Q
EOS L
p (GeV/fm^3)
2
0
EOS
$n_B=0$
0
1
2
3
4
5
6
e (GeV/fm^3)

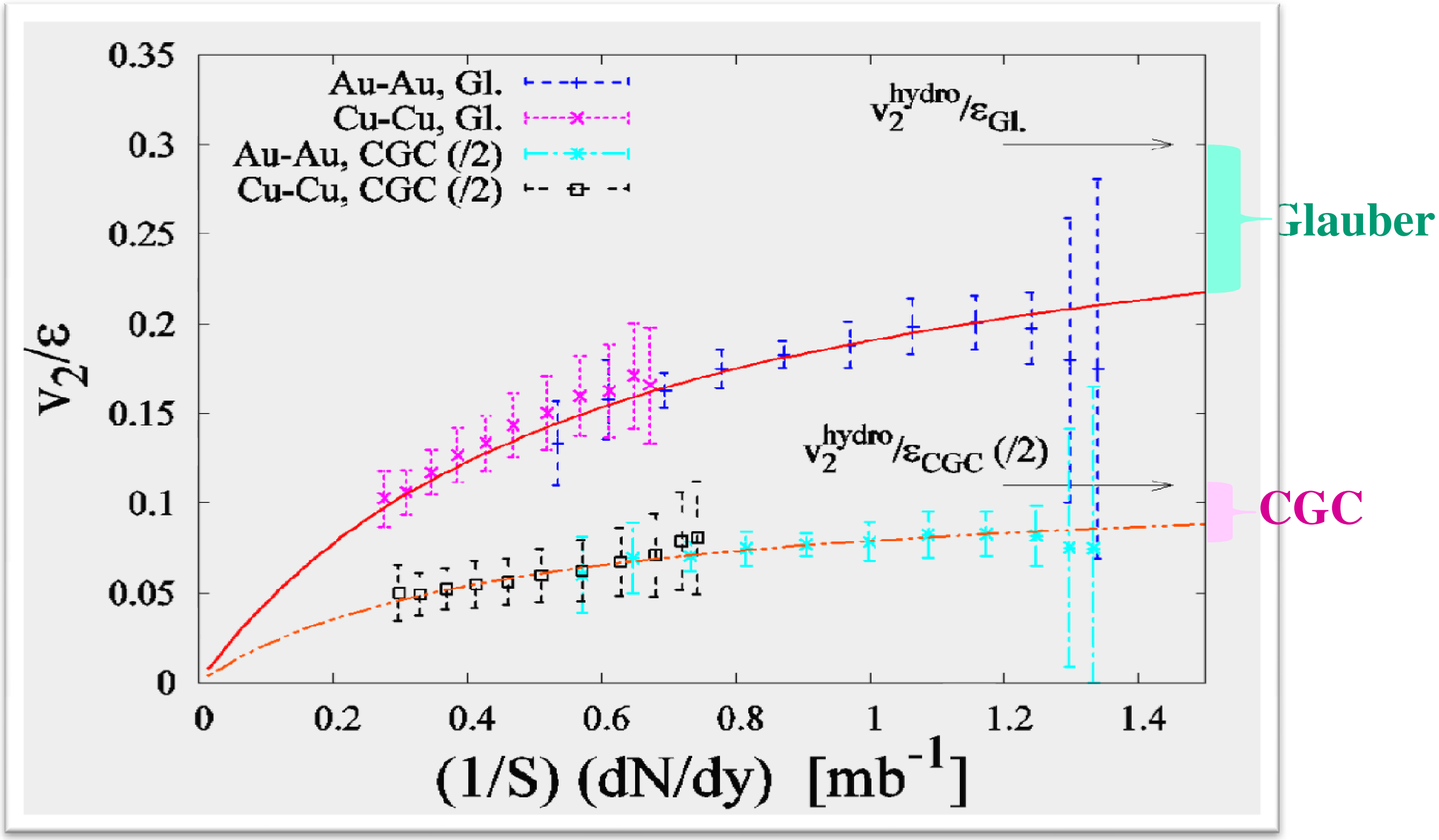


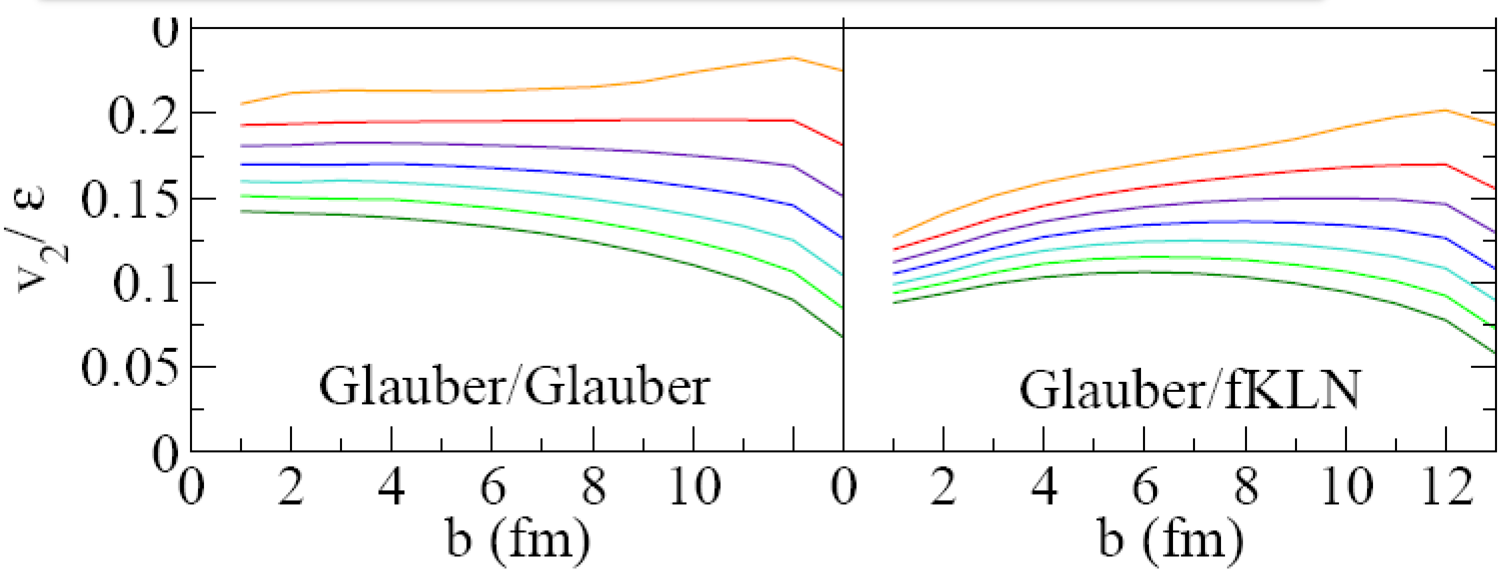
-To describe the experimental data, one need to consider the contribution from all kinds of aspects besides shear viscosity

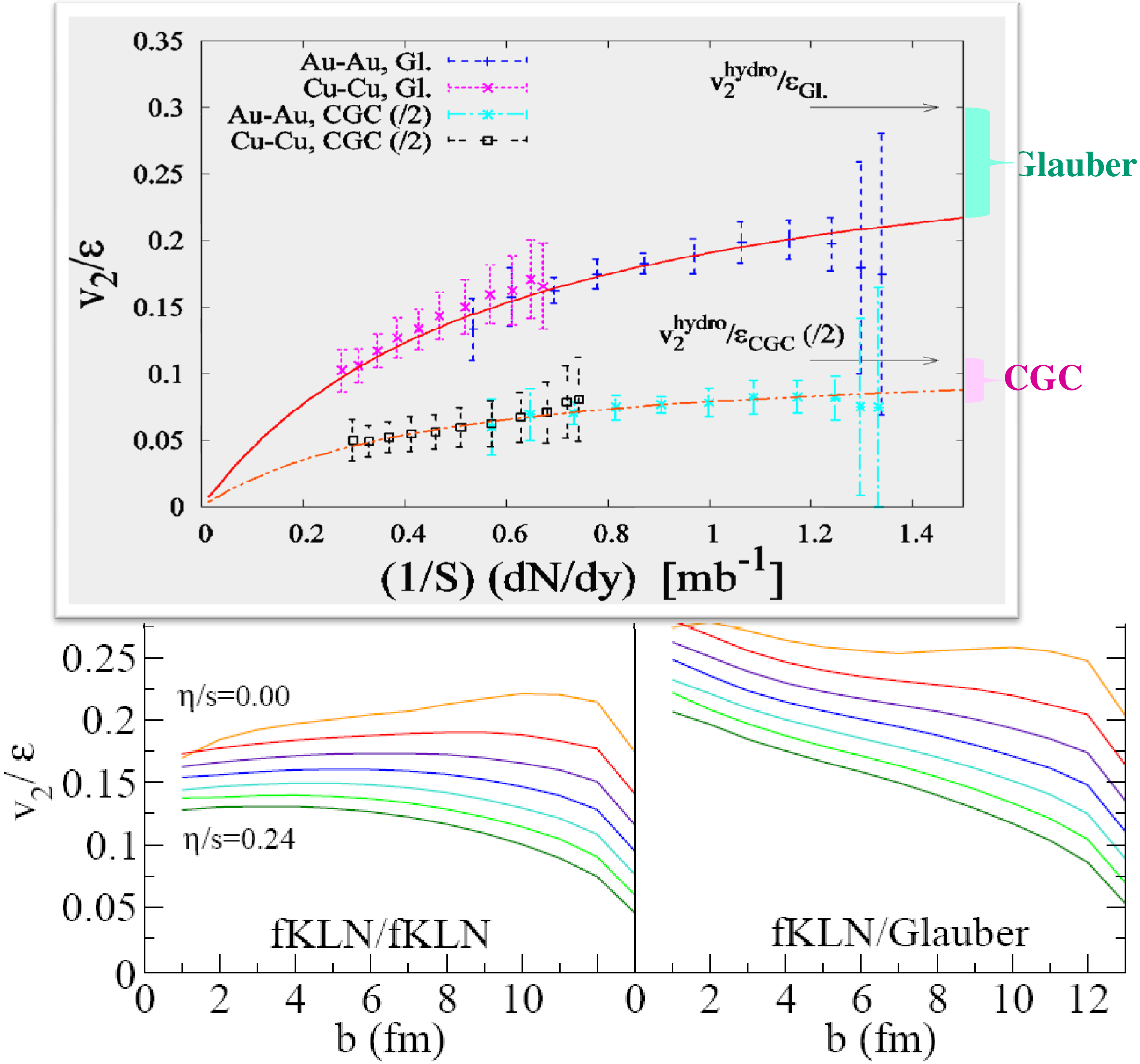
Effects from different initializations

-Heinz, Moreland and Song




Au-Au, Gl.
Cu-Cu, Gl.
Au-Au, CGC (/2)
Cu-Cu, CGC (/2)
$v_2^{hydro}/\varepsilon_{Gl.}$
$v_2^{hydro}/\varepsilon_{CGC}$ (/2)
Glauber
CGC
v_2/ε
(1/S) (dN/dy) [mb^{-1}]


Glauber/Glauber
Glauber/fKLN
v_2/ε
b (fm)
b (fm)



Au-Au, Gl.
Cu-Cu, Gl.
Au-Au, CGC (/2)
Cu-Cu, CGC (/2)
$v_2^{hydro}/\varepsilon_{Gl.}$
$v_2^{hydro}/\varepsilon_{CGC}$ (/2)
Glauber
CGC
v_2/ε
(1/S) (dN/dy) [mb^{-1}]
η/s=0.00
η/s=0.24
fKLN/fKLN
fKLN/Glauber
b (fm)
b (fm)